

Name: Solutions

\_\_\_\_\_ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with**  $f'(x) =$ ,  $dy/dx =$ , or similar.
- Circle your final answer.

Compute the derivatives of the following functions.

$$1. g(x) = \frac{2x^2 - x^3 + 4x^{1/2}}{x^{1/2}} = 2x^{3/2} - x^{5/2} + 4$$

$$g'(x) = 2\left(\frac{3}{2}\right)x^{1/2} - \frac{5}{2}x^{3/2}$$

$$g'(x) = 3x^{1/2} - \frac{5}{2}x^{3/2}$$

$$2. r(\theta) = \frac{1}{\sin(\theta)} = \csc(\theta)$$

$$r'(\theta) = -\cot\theta \csc\theta$$

$$r'(\theta) = \frac{\sin\theta(0) - 1(\cos\theta)}{\sin^2\theta}$$

$$= -\frac{\cos\theta}{\sin^2\theta} = -\cot\theta \csc\theta$$

$$3. f(x) = \sqrt{6} - \frac{1}{x^3}$$

$$f'(x) = \frac{3}{x^4}$$

4.  $y = ax^3 + e^{bx^2}$ , where  $a$  and  $b$  are fixed constants

$$y' = 3ax^2 + 2bx e^{bx^2}$$

5.  $s(t) = \tan(\ln(-t^2))$

$$s'(t) = \sec^2(\ln(-t^2)) \cdot \frac{1}{-t^2} (-2t)$$

$$= \frac{2}{t} \sec^2(\ln(-t^2))$$

6.  $g(x) = \left(\frac{1}{x} - x^2\right)^3 (2x - 1)$

$$g'(x) = \left(\frac{1}{x} - x^2\right)^3 (2) + 3\left(\frac{1}{x} - x^2\right)^2 \left(-\frac{1}{x^2} - 2x\right) (2x - 1)$$

7.  $h(y) = (\ln(y) + y)^{5/4}$

$$h'(y) = \frac{5}{4} (\ln(y) + y)^{1/4} \left( \frac{1}{y} + 1 \right)$$

8.  $f(x) = \frac{\cos(\pi x)}{e^{2x} - 1}$

$$f'(x) = \frac{(e^{2x} - 1)(-\sin(\pi x)\pi) - \cos(\pi x) \cdot 2e^{2x}}{(e^{2x} - 1)^2}$$

9.  $y = \ln(x) \tan(3x) \cos(x - \pi)$

$$y' = \frac{1}{x} \tan(3x) \cos(x - \pi) + \ln(x) \left[ 3 \sec^2(3x) \cos(x - \pi) + \tan(3x) (-\sin(x - \pi)) \right]$$

10.  $f(x) = \ln(e^x + \ln(3))$

$$f'(x) = \frac{1}{e^x + \ln(3)} \cdot e^x$$

11.  $f(x) = (\sqrt{1-x^2}) \arcsin(x)$

$$f'(x) = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) \arcsin x + \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{-x \arcsin x}{\sqrt{1-x^2}} + 1$$

12. Compute  $dy/dx$  if  $x^2 - 3 = e^y + xy^2$ . You must solve for  $dy/dx$ .

$$2x = e^y \frac{dy}{dx} + y^2 + 2xy \frac{dy}{dx}$$

$$2x - y^2 = (e^y + 2xy) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - y^2}{e^y + 2xy}$$