Math 251: Integral Proficiency **24 April 2019**

Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Be sure to include constants of integration when appropriate.
- Circle your final answer.

Compute the following integrals.

1. $\int_{\pi}^{2\pi} (\cos\theta - 1)\, d\theta$ $= \sin\theta - \theta \Big|_{\pi}^{2\pi} = (\sin(2\pi) - 2\pi) - (\sin\pi - \pi)$

$= -\pi$

2. $\int \frac{4 - 2\ln t}{t}\, dt$ $= \int (4 - 2u)\, du = 4u - u^2 + C = 4\ln t - (\ln t)^2 + C$

$u = \ln t$

$du = \frac{1}{t}\, dt$

3. $\int_1^2 \frac{x^3 - 1}{x^2}\, dx$ $= \int_1^2 x - \frac{1}{x^2}\, dx = \frac{x^2}{2} + \frac{1}{x}\Big|_1^2 = \left(2 + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right)$

$= 1$

4. $\int \tan^2 x \sec^2 x\, dx$ $= \int u^2\, du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

$u = \tan x$

$du = \sec^2 x\, dx$

5. $\int \frac{1+x^2}{2} + \frac{2}{1+x^2}\, dx$ $= \int \left(\frac{1}{2} + \frac{x^2}{2}\right) dx + 2\arctan x$

$= \frac{1}{2}x + \frac{x^3}{6} + 2\arctan x + C$

6. $\int z\sqrt{3-z}\, dz$ $= -\int (3-u)\sqrt{u}\, du = -\int \left(3u^{1/2} - u^{3/2}\right) du$

$u = 3 - z$

$du = -dz$

$= -2u^{3/2} + \frac{2}{5}u^{5/2} + C$

$z = 3 - u$

$= -2(3-z)^{3/2} + \frac{2}{5}(3-z)^{5/2} + C$

7. $\int (\sin\theta)e^{\cos\theta}\,d\theta$ $= -\int e^u\,du = -e^u + C$

$u = \cos\theta$

$du = -\sin\theta\,d\theta$ $= -e^{\cos\theta} + C$

8. $\int_{-1}^{1} (x+3)(x-2)\,dx$ $= \int_{-1}^{1} (x^2 + x - 6)\,dx$ $= \frac{x^3}{3} + \frac{x^2}{2} - 6x \Big|_{-1}^{1}$

$= \left(\frac{1}{3} + \frac{1}{2} - 6\right) - \left(\frac{-1}{3} + \frac{1}{2} + 6\right)$

$= \frac{2}{3} - 12$

$= \frac{-34}{3}$

9. $\int t\cos(2-5t^2)\,dt$ $= \frac{-1}{10}\int \cos u\,du = -\frac{1}{10}\sin u + C$

$u = 2 - 5t^2$

$du = -10t\,dt$ $= -\frac{1}{10}\sin(2-5t^2) + C$

$-\frac{1}{10}\,du = t\,dt$

10. $\int \sqrt[3]{x^2} - \sqrt[3]{4}\,dx$ $= \int (x^{2/3} - \sqrt[3]{4})\,dx = \frac{3}{5}x^{5/3} - \sqrt[3]{4}\,x + C$

11. $\int \left(7e^w - \frac{1}{w^3}\right) dw$ $= 7e^w + \frac{1}{2w^2} + C$

12. $\int \frac{t^2}{t^3-2}\,dt$ $= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$u = t^3 - 2$

$du = 3t^2\,dt$

$= \frac{1}{3}\ln|t^3-2| + C$