Name: Solutions _____ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Be sure to include constants of integration when appropriate.
- Circle your final answer.

Compute the following integrals.

1. $\int_1^2 \frac{x^4+1}{x^3}\,dx = \int_1^2 x + x^{-3}\,dx = \frac{x^2}{2} + \frac{x^{-2}}{-2}\Big|_1^2$

$= \left(2 - \frac{1}{8}\right) - \left(\frac{1}{2} - \frac{1}{2}\right) = \boxed{\frac{15}{8}}$

2. $\int \frac{2-3\ln t}{t}\,dt$

$u = \ln t$

$du = \frac{1}{t}\,dt$

$= \int (2-3u)\,du = 2u - \frac{3}{2}u^2 + C$

$= \boxed{2\ln t - \frac{3}{2}(\ln t)^2 + C}$

3. $\int_\pi^{2\pi} (\cos\theta - 4)\,d\theta = -\sin\theta - 4\theta\Big|_\pi^{2\pi}$

$= (\sin 2\pi - 8\pi) - (\sin\pi - 4\pi)$

$= -8\pi + 4\pi = \boxed{-4\pi}$

4. $\int z\sqrt{z+2}\,dz$ $= \int (u-2)\sqrt{u}\,du = \int u^{3/2} - 2u^{1/2}\,du$

$u = z+2$

$du = dz$

$u-2 = z$

$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$

$= \frac{2}{5}(z+2)^{5/2} - \frac{4}{3}(z+2)^{3/2} + C$

5. $\int \tan^2 x \sec^2 x\,dx$ $= \int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

$u = \tan x$

$du = \sec^2 x\,dx$

6. $\int \frac{4}{1+x^2} + \frac{1+x^2}{4}\,dx$ $= 4\arctan x + \int \left(\frac{1}{4} + \frac{x^2}{4}\right)dx$

$= 4\arctan x + \frac{1}{4}x + \frac{x^3}{12} + C$

7. $\int t\cos(5-3t^2)\,dt$ $= -\frac{1}{6}\int \cos u\,du = -\frac{1}{6}\sin u + C$

$u = 5-3t^2$

$du = -6t\,dt$

$-\frac{1}{6}du = t\,dt$

$= -\frac{1}{6}\sin(5-3t^2) + C$

8. $\int (\sin\theta)e^{\cos\theta}\,d\theta$ $= -\int e^u\,du = -e^u + C = -e^{\cos\theta} + C$

$u = \cos\theta$

$du = -\sin\theta\,d\theta$

9. $\int_{-1}^{1}(x+3)(x-4)\,dx$ $= \int_{-1}^{1}(x^2-x-12)\,dx = \left(\frac{x^3}{3} - \frac{x^2}{2} - 12x\right)\Big|_{-1}^{1}$

$= \left(\frac{1}{3} - \frac{1}{2} - 12\right) - \left(\frac{-1}{3} - \frac{1}{2} + 12\right)$

$= \frac{2}{3} - 24$

$= \frac{-70}{3}$

10. $\int \frac{t^2}{t^3-9}\,dt$ $= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$u = t^3 - 9$

$du = 3t^2\,dt$

$\frac{1}{3}du = t^2\,dt$

$= \frac{1}{3}\ln|t^3-9| + C$

11. $\int \sqrt[3]{x^4} - \sqrt[3]{5}\,dx$ $= \int \left(x^{4/3} - \sqrt[3]{5}\right)dx$ $= \frac{3}{7}x^{7/3} - \sqrt[3]{5}x + C$

12. $\int \left(3e^w - \frac{1}{w^5}\right)dw$ $= 3e^w + \frac{1}{4}w^{-4} + C$