

Name: Solutions

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- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Be sure to include constants of integration when appropriate.
- Circle your final answer.

Compute the following integrals.

$$\begin{aligned}
 1. \int_0^3 x(x+2) dx &= \int_0^3 x^2 + 2x dx \\
 &= \left. \frac{1}{3}x^3 + x^2 \right|_0^3 \\
 &= \frac{1}{3}(3)^3 + 3^2 = \boxed{18}
 \end{aligned}$$

$$\begin{aligned}
 2. \int \sin(\pi\theta - 1) d\theta \\
 &= -\frac{\cos(\pi\theta - 1)}{\pi} + C
 \end{aligned}$$

$$\begin{aligned}
 3. \int xe^{(1-x^2)} dx & \quad u = 1 - x^2 \\
 & \quad du = -2x dx \\
 &= -\frac{1}{2} \int e^u du \\
 &= -\frac{1}{2} e^u + C = -\frac{1}{2} e^{1-x^2} + C
 \end{aligned}$$

$$4. \int \frac{x^2}{2+x^3} dx$$

$$u = 2+x^3$$

$$du = 3x^2 dx$$

$$= \frac{1}{3} \int \frac{du}{u}$$

$$= \frac{1}{3} \ln|u| + C = \boxed{\frac{1}{3} \ln|2+x^3| + C}$$

$$5. \int_1^3 \frac{2+x^3}{x^2} dx$$

$$= \int_1^3 (2x^{-2}) + x dx$$

$$= \left. -\frac{2}{x} + \frac{1}{2}x^2 \right|_1^3$$

$$= \left(-\frac{2}{3} + \frac{1}{2}(3)^2 \right) - \left(-2 + \frac{1}{2} \right) = \boxed{\frac{16}{3}}$$

$$6. \int \frac{3}{\sqrt{1-x^2}} dx$$

$$= \boxed{3 \arcsin x + C}$$

$$7. \int \frac{2\sqrt{\ln(t)+2}}{t} dt$$

$$u = \ln t + 2$$

$$du = \frac{1}{t} dt$$

$$= 2 \int u^{1/2} du$$

$$= 2 \left(\frac{2}{3} \right) u^{3/2} + C$$

$$= \frac{4}{3} (\ln t + 2)^{3/2} + C$$

$$8. \int_0^1 (e^x + x^e) dx$$

$$= e^x + \frac{1}{e+1} x^{e+1} \Big|_0^1$$

$$= e^1 + \frac{1}{e+1} (1)^{e+1} - \left(e^0 + \frac{1}{e+1} (0)^{e+1} \right)$$

$$= e + \frac{1}{e+1} - 1$$

$$9. \int \frac{w}{\sqrt{w+1}} dw$$

$$u = w + 1 \quad w = u - 1$$

$$du = dw$$

$$= \int \frac{u-1}{\sqrt{u}} du$$

$$= \frac{2}{3} u^{3/2} - 2 u^{1/2} + C$$

$$= \int (u^{1/2} - u^{-1/2}) du$$

$$= \frac{2}{3} (w+1)^{3/2} - 2 (w+1)^{1/2} + C$$

10. $\int \frac{\sec^2 x}{\tan^3 x} dx$

$$u = \tan x$$

$$du = \sec^2 x dx$$

$$= \int \frac{du}{u^3} = -\frac{1}{2} u^{-2} + C$$

$$= -\frac{1}{2 \tan^2 x} + C$$

11. $\int \left(\ln(2) - \frac{1}{z} \right) dz$

$$= \ln(2) \cdot z - \ln|z| + C$$

12. $\int \frac{\sin t}{\cos t} dt$

$$u = \cos t$$

$$du = -\sin t dt$$

$$= - \int \frac{du}{u}$$

$$= -\ln|u| + C$$

$$= -\ln|\cos t| + C$$