

SOLUTIONS

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- Circle your final answer.

Compute the derivatives of the following functions.

1. $f(x) = \pi x^{1/3} - 2e^x + \ln 7$

$$f'(x) = \frac{\pi}{3} x^{-2/3} - 2e^x$$

2. $y = (x - x^2) \sin(x)$

$$\frac{dy}{dx} = (1 - 2x) \sin(x) + (x - x^2) \cos(x)$$

3. $f(t) = \frac{t^2 - t + 4t^{1/2}}{t^{1/2}} = t^{3/2} - t^{1/2} + 4$

$$f'(t) = \frac{3}{2} t^{1/2} - \frac{1}{2} t^{-1/2}$$

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4. $f(t) = b + t^2 \ln(at)$

$$f'(t) = 2t \ln(at) + t^2 \cdot \frac{1}{at} \cdot a$$

5. $f(x) = \frac{1}{\cos(x)} = \sec(x)$

$$f'(x) = \sec(x) \tan(x)$$

6. $f(x) = \frac{\cos(x)}{1 + \sin(3x)}$

$$f'(x) = \frac{(-\sin(x))(1 + \sin(3x)) - \cos(x)(\cos(3x) \cdot 3)}{(1 + \sin(3x))^2}$$

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7. $f(x) = \sec(x)x^{1/3}e^{4x}$

$$f'(x) = \sec(x)\tan(x) \times \frac{1}{3} e^{4x} \\ + \sec(x) \frac{1}{3} x^{-2/3} \cdot e^{4x} \\ + \sec(x) \times \frac{1}{3} \cdot e^{4x} \cdot 4$$

8. $f(z) = \arctan(\sqrt{z} + \sqrt{5})$

$$f'(z) = \frac{1}{1 + (\sqrt{z} + \sqrt{5})^2} \left(\frac{1}{2} z^{-1/2} \right)$$

9. $f(t) = \tan(\ln(t^3 - 1))$

$$f'(t) = \sec^2(\ln(t^3 - 1)) \cdot \frac{1}{t^3 - 1} \cdot 3t^2$$

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10. $f(x) = \frac{1}{7x^2} + \left(\pi \frac{x-5}{4}\right)^3$

$$f'(x) = \frac{1}{7}(-2)x^{-3} + 3\left(\pi \frac{x-5}{4}\right)^2 \cdot \frac{\pi}{4}$$

11. $f(x) = \cos^5(x^2 - x)$

$$f'(x) = 5 \cos^4(x^2 - x) (-\sin(x^2 - x)(2x - 1))$$

12. Compute dy/dx if $x^2y - 3 = e^y \sin(x)$. You must solve for dy/dx .

$$2x \cdot y + x^2 \cdot y' = e^y y' \cdot \sin(x) + e^y \cdot \cos(x)$$

$$\frac{dy}{dx} = y' = \frac{e^y \cos(x) - 2xy}{x^2 - e^y \sin(x)}$$