Name: SOLUTIONS ______ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Be sure to include constants of integration when appropriate.
- Circle your final answer.

Compute the following integrals.

1. $\int_1^4 \left(\frac{1}{x} - \sqrt{x}\right)\,dx$

$$= \ln|x| - \frac{2}{3}x^{3/2}\Big]_1^4$$

$$= \left(\ln 4 - \frac{2}{3}4^{3/2}\right) - \left(\ln 1 - \frac{2}{3}\cdot 1^{3/2}\right)$$

$$= 2\ln 2 - \frac{16}{3} - 0 + \frac{2}{3} = 2\ln 2 - \frac{14}{3}$$

2. $\int \left(7^{\frac{1}{3}} + e^{5x} - \pi x^2\right)\,dx$

$$= 7^{\frac{1}{3}}x + \frac{1}{5}e^{5x} - \frac{\pi}{3}x^3 + C$$

3. $\int \frac{1}{x\ln(x)}\,dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C$$

$$\left[\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}\right]$$

$$= \ln|\ln x| + C$$

4. $\int (x-2)(x-3)\,dx$ $= \int x^2 - 5x + 6\,dx$

$= \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x + C$

5. $\int \sec^2(x)\,e^{\tan(x)}\,dx$ $=$ $\int e^u\,du$

$u = \tan(x)$
$du = \sec^2 x\,dx$

$= e^u + C$

$= e^{\tan(x)} + C$

6. $\int \left(\frac{8x}{1-x^2} + \cos(x)\right)\,dx$ $=$ $\int \frac{8 \cdot \frac{du}{-2}}{u} + \int \cos(x)\,dx$

$u = 1-x^2$
$du = -2x\,dx$
$\frac{du}{-2} = x\,dx$

$= -4\ln|u| + \sin(x) + C$

$= -4\ln|1-x^2| + \sin(x) + C$

7. $\int x\sqrt{x-9}\,dx$ $= \int (u+9)\sqrt{u}\,du = \int u^{3/2} + 9u^{1/2}\,du$

$\left[\begin{array}{l} u = x-9 \\ x = u+9 \\ du = dx \end{array}\right]$

$= \frac{2}{5}u^{5/2} + 9\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x-9)^{5/2} + 6(x-9)^{3/2} + C$

8. $\int \cos(x)\,(\sin(x)-3)^5\,dx$ $= \int u^5\,du$

$\left[\begin{array}{l} u = \sin(x)-3 \\ du = \cos(x)\,dx \end{array}\right]$

$= \frac{1}{6}u^6 + C$

$= \frac{1}{6}(\sin(x)-3)^6 + C$

9. $\int \sec^2\left(\frac{\pi}{2}t\right)\,dt$ $= \frac{2}{\pi}\int \sec^2(u)\,du$

$\left[\begin{array}{l} u = \frac{\pi}{2}t \\ du = \frac{\pi}{2}dt \end{array}\right]$

$\frac{2}{\pi}du = dt$

$= \frac{2}{\pi}\tan(u) + C$

$= \frac{2}{\pi}\tan\left(\frac{\pi}{2}t\right) + C$

10. $\int \frac{6}{\sqrt{1-s^2}}\,ds$ $= 6\arcsin(s) + C$

11. $\int_{-1}^{0} e^{-8t+5}\,dt$ $=$

$$\left[\begin{array}{l} u = -8t+5 \\ du = -8\,dt \\ \frac{du}{-8} = dt \end{array}\right]$$

$$\int_{13}^{5} e^{u}\,\frac{du}{-8} = -\frac{1}{8}\,e^{u}\Big]_{13}^{5}$$

$$= \frac{1}{8}\,e^{u}\Big]_{5}^{13} = \frac{1}{8}\left(e^{13} - e^{5}\right)$$

12. $\int \frac{2x^3-5}{x}\,dx$ $= \int 2x^2 - 5\,\frac{1}{x}\,dx$

$$= \frac{2}{3}x^3 - 5\ln|x| + C$$