

Name: Solutions

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- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

$$\text{a. } f(x) = \frac{7^{1/3}}{x^{1/3}} + e^{x-1} + \pi^2 = 7^{1/3} x^{-1/3} + e^{x-1} + \pi^2$$

$$f'(x) = 7^{1/3} \left(-\frac{1}{3} x^{-4/3} \right) + e^{x-1}$$

$$\text{b. } f(x) = \frac{\cos(x)}{\sin(x)} = \cot x$$

$$f'(x) = -\csc^2 x$$

$$\text{c. } f(x) = (x^5 - x) \cos(x)$$

$$f'(x) = (5x^4 - 1) \cos x + (x^5 - x) (-\sin x)$$

d. $f(x) = \frac{1 + e^{-11x}}{\tan(x)}$

$$f'(x) = \frac{(\tan x)(-11e^{-11x}) - (1 + e^{-11x})(\sec^2 x)}{\tan^2 x}$$

e. $f(t) = \frac{t\sqrt{t} - 9\sqrt{t} + 1}{\sqrt{t}} = t - 9 + t^{-1/2}$

$$f'(t) = 1 - \frac{1}{2} t^{-3/2}$$

f. $f(t) = t^p \ln(at + 1)$

$$f'(t) = (pt^{p-1}) \ln(at+1) + t^p \left(\frac{a}{at+1} \right)$$

g. $f(x) = 2^x \sin(2x)$

$$f'(x) = (\ln 2) 2^x \sin(2x) + 2 \cdot 2^x \cos(2x)$$

h. $f(x) = \frac{1}{5x} + \left(\frac{\pi(x+1)}{4}\right)^3 = \frac{1}{5} x^{-1} + \left(\frac{\pi}{4}(x+1)\right)^3$

$$f'(x) = -\frac{1}{5} x^{-2} + 3\left(\frac{\pi}{4}(x+1)\right)^2 \left(\frac{\pi}{4}\right)$$

i. $f(t) = \ln(x + \sec^2(x))$

$$f'(t) = \frac{1 + 2 \sec x \sec x \tan x}{x + \sec^2 x}$$

$$\text{j. } f(x) = \sin\left(\frac{x}{e^x}\right) = \sin(xe^{-x})$$

$$f'(x) = \left(\cos\left(\frac{x}{e^x}\right)\right) (1 \cdot e^{-x} - x e^{-x})$$

$$\text{k. } f(z) = \arcsin\left(\frac{1}{z}\right) = \arcsin(z^{-1})$$

$$f'(z) = \frac{-z^{-2}}{\sqrt{1-z^{-2}}}$$

l. Compute dy/dx if $e^y + \cos x = \ln(5) - xy$. You must solve for dy/dx .

$$e^y \frac{dy}{dx} - \sin x = -y - x \frac{dy}{dx}$$

$$\frac{dy}{dx} (e^y + x) = \sin x - y$$

$$\frac{dy}{dx} = \frac{\sin x - y}{e^y + x}$$