

Name: _____ **Solutions** _____ Class (circle): Sync. Online

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression; a correct answer for a non-trivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = x^{2/3} + x^{-2} + \pi^2$

$$f'(x) = \frac{2}{3}x^{-1/3} - 2x^{-3}$$

b. $r(\theta) = \frac{2}{\cos \theta}$

$$r'(\theta) = -\frac{2}{\cos^2(\theta)} \cdot (-\sin \theta)$$

c. $h(t) = (2t^3 - t)(4 + 8t)$

$$h'(t) = (6t^2 - 1)(4 + 8t) + (2t^3 - t) \cdot 8$$

d. $g(x) = e^{2x} \ln(x) \cos(x)$

$$g'(x) = (e^{2x} \cdot 2 \cdot \ln(x) + e^{2x} \cdot \frac{1}{x}) \cos(x) + e^{2x} \cdot \ln(x) \cdot (-\sin(x))$$

e. $w(r) = (r^3 - 1) \cdot \arcsin(r^2)$

$$w'(r) = 3r^2 \cdot \arcsin(r^2) + (r^3 - 1) \cdot \frac{1}{\sqrt{1-r^4}} \cdot 2r$$

f. $y = \frac{e^{-x}}{2 + \sin(bx)}$, where b is a fixed constant

$$y'(x) = \frac{-e^{-x}(2 + \sin(bx)) - e^{-x} \cdot \cos(bx) \cdot b}{(2 + \sin(bx))^2}$$

g. $k(x) = \frac{xe^x}{1+x}$

$$k'(x) = \frac{(e^x + xe^x)(1+x) - xe^x \cdot 1}{(1+x)^2}$$

h. $f(x) = \ln(\sqrt{2} + \sec(x))$

$$f'(x) = \frac{1}{\sqrt{2} + \sec(x)} \cdot \sec(x) \cdot \tan(x)$$

i. $y = \left(\frac{1}{x} + \frac{5x^3}{2}\right)^5$

$$y'(x) = 5 \left(\frac{1}{x} + \frac{5x^3}{2}\right)^4 \cdot \left(-\frac{1}{x^2} + \frac{15x^2}{2}\right)$$

j. $s(t) = \sin(\sqrt{t+t^4})$

$$s'(t) = \cos(\sqrt{t+t^4}) \cdot \frac{1}{2\sqrt{t+t^4}} \cdot (1+4t^3)$$

k. $g(\theta) = \tan\left(\frac{2}{\theta^3} + e\right)$

$$g'(\theta) = \sec^2\left(\frac{2}{\theta^3} + e\right) \cdot \left(-\frac{6}{\theta^4}\right)$$

l. Compute dy/dx if $x^2y - e^x = 2 + \cos(y)$. You must solve for dy/dx .

$$\frac{d}{dx}(x^2y - e^x) = \frac{d}{dx}(2 + \cos(y))$$

$$2xy + x^2 \cdot y' - e^x = -\sin(y) \cdot y'$$

$$y'(x^2 + \sin(y)) = e^x - 2xy$$

$$y' = \frac{e^x - 2xy}{x^2 + \sin(y)}$$