**Name:** Solutions **Instructor (circle):** **Maxwell** **Jurkowski** **Sus**

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

**1. [12 points]** Compute the following definite/indefinite integrals.

**a.** $\int_0^\pi (5e^x + 3\sin(x))\, dx = \left(5e^x - 3\cos(x)\right)\Big|_0^\pi =$

$$= \left(5e^\pi - 3\cos(\pi)\right) - \left(5 - 3\cdot\cos(0)\right) =$$

$$= 5e^\pi + 3 - 5 + 3 = \boxed{5e^\pi + 1}$$

**b.** $\int \frac{(1+x)^2}{2x}\, dx = \int \frac{1+2x+x^2}{2x}\, dx = \int \left(\frac{1}{2x} + 1 + \frac{x}{2}\right) dx =$

$$= \boxed{\frac{1}{2}\ln|x| + x + \frac{x^2}{4} + C}$$

**c.** $\int (x^2 - 3\ln 2)\, dx = \boxed{\frac{x^3}{3} - 3\ln 2\, x + C}$

**d.** $\int \sec\left(\frac{\pi x}{2}\right)\tan\left(\frac{\pi x}{2}\right)\,dx = \frac{2}{\pi}\int \sec(u)\tan(u)\,du =$

$\frac{\pi x}{2} = u$

$du = \frac{\pi}{2}\,dx$

$dx = \frac{2}{\pi}\,du$

$= \frac{2}{\pi}\sec(u) + C =$

$= \boxed{\frac{2}{\pi}\sec\left(\frac{\pi x}{2}\right) + C}$

**e.** $\int \frac{(\arctan(x))^2}{x^2+1}\,dx = \int \frac{u^2}{\cancel{1+x^2}}\,(\cancel{1+x^2})\,du = \frac{u^3}{3} + C =$

$\arctan(x) = u$

$\frac{1}{1+x^2}\,dx = du$

$dx = (1+x^2)\,du$

$= \boxed{\frac{(\arctan(x))^3}{3} + C}$

**f.** $\int \sqrt{x}(x^2+3x+2)\,dx = \int \left(x^{5/2} + 3x^{3/2} + 2x^{1/2}\right)dx =$

$\boxed{= \frac{x^{7/2}}{7/2} + 3\,\frac{x^{5/2}}{5/2} + 2\,\frac{x^{3/2}}{3/2} + C}$

**g.** $\int \left(2\sec^2(x) + \frac{1}{\sqrt{1-x^2}}\right)\,dx$ $=$ $\boxed{2\tan(x) + \arcsin(x) + C}$

**h.** $\int x\sqrt{x+5}\,dx$ $= \int x\sqrt{u}\,du = \int (u-5)\sqrt{u}\,du =$

$u = x+5$

$du = dx$

$= \int \left(u^{3/2} - 5u^{1/2}\right)du =$

$= \frac{u^{5/2}}{5/2} - 5\frac{u^{3/2}}{3/2} + C =$

$= \boxed{\frac{(x+5)^{5/2}}{5/2} - 5\frac{(x+5)^{3/2}}{3/2} + C}$

**i.** $\int \frac{\sec^2(x)}{\tan^2(x)}\,dx$ $=$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$dx = \frac{1}{\sec^2(x)}du$

$= \int \frac{\cancel{\sec^2(x)}}{u^2}\frac{1}{\cancel{\sec^2(x)}}du = \int \frac{1}{u^2}du = -\frac{1}{u} + C = \boxed{-\frac{1}{\tan(x)} + C}$

**j.** $\int \frac{\cos(\ln x)}{x}\,dx = \int \frac{\cos(u)}{\cancel{x}} \cancel{x}\,du = \sin(u) + C = \boxed{\sin(\ln(x)) + C}$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$dx = x\,du$

**k.** $\int \frac{6x^2}{x^3+1}\,dx = \int \frac{6\cancel{x^2}}{u} \frac{1}{3\cancel{x^2}}\,du = 2\ln|u| + C = \boxed{2\ln|1+x^3| + C}$

$x^3 + 1 = u$

$3x^2\,dx = du$

$dx = \frac{1}{3x^2}\,du$

**l.** $\int (x-1)e^{(x-1)^2}\,dx = \int \cancel{(x-1)}e^u \frac{1}{2\cancel{(x-1)}}\,du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{(x-1)^2} + C}$

$u = (x-1)^2$

$du = 2(x-1)\,dx$

$dx = \frac{1}{2(x-1)}\,du$