Name: Solutions

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int (4x^3 + \cos(x))\, dx$ $= x^4 + \sin(x) + C$

b. $\int \sin(2x)\, dx$ $= -\frac{1}{2}\cos(2x) + C$

c. $\int_1^2 (xe^{x^2})\, dx$ $= \frac{1}{2}\int_1^4 e^u\, du = \frac{1}{2} e^u \Big]_1^4 = \frac{1}{2}(e^4 - e)$

let $u = x^2$ if $x=1,\ u = 1^2 = 1$

$du = 2x\,dx$ $x = 2,\ u = 2^2 = 4$

$\frac{1}{2} du = x\,dx$

d. $\int \left(\frac{x}{2}+\frac{2}{x}+\frac{2}{3}\right)\,dx$ $= \frac{1}{4}x^2 + 2\ln|x| + \frac{2}{3}x + C$

e. $\int \frac{1-\sin(2x)}{2x+\cos(2x)}\,dx$ $= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

let $u = 2x + \cos(2x)$ $= \frac{1}{2}\ln|2x+\cos(2x)| + C$

$du = [2 - 2\sin(2x)]\,dx$

$\frac{1}{2}du = (1 - \sin(2x))\,dx$

f. $\int \frac{5}{x(\ln x)^2}\,dx$ $= 5\int (\ln(x))^{-2}\left(\frac{dx}{x}\right) = 5\int u^{-2}\,du$

let $u = \ln(x)$ $= -5u^{-1} + C$

$du = \frac{1}{x}dx$

$= -5(\ln(x))^{-1} + C$

g. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ $= \arcsin(x) + C$

h. $\int \frac{x^3}{\sqrt{4-x^4}}\,dx$ $= \int (4-x^4)^{-\frac{1}{2}} x^3 dx = -\frac{1}{4}\int u^{-\frac{1}{2}} du = -\frac{1}{2} u^{\frac{1}{2}} + C$

let $u = 4-x^4$

$du = -4x^3 dx$

$-\frac{1}{4} du = x^3 dx$

$= -\frac{1}{2}(4-x^4)^{\frac{1}{2}} + C$

i. $\int (e^{-x} + \sec^2(x))\,dx$ $= -e^{-x} + \tan(x) + C$

j. $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C$

let $u = \tan^{-1}x$

$du = \frac{dx}{1+x^2}$

$= \frac{1}{2}(\tan^{-1}x)^2 + C$

k. $\int_{-1}^{1} x(3-x)\,dx$ $= \int_{-1}^{1}(3x - x^2)\,dx = \frac{3}{2}x^2 - \frac{1}{3}x^3\Big]_{-1}^{1} = \left(\frac{3}{2}(1)^2 - \frac{1}{3}(1)^3\right) - \left(\frac{3}{2}(-1)^2 - \frac{1}{3}(-1)^3\right)$

$= \left(\frac{3}{2} - \frac{1}{3}\right) - \left(\frac{3}{2} + \frac{1}{3}\right) = \frac{-2}{3}$

l. $\int \frac{x}{x+1}\,dx$ $= \int \frac{u-1}{u}\,du = \int\left(1 - \frac{1}{u}\right)du = u - \ln|u| + C$

let $u = x+1$

$du = dx$

$u - 1 = x$

$= x+1 - \ln|x+1| + C$