Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $y = 3\sec(3x)$

$$y' = 3\sec(3x)\tan(3x)(3)$$
$$= 9\sec(3x)\tan(3x)$$

b. $f(x) = \dfrac{\sqrt{x}}{6} + \dfrac{5}{\sqrt{x}} - \dfrac{4}{\sqrt{5}}$ $= \frac{1}{6}x^{\frac{1}{2}} + 5x^{-1/2} - \frac{4}{\sqrt{5}}$

$$f'(x) = \frac{1}{6}\cdot\frac{1}{2}x^{-1/2} + 5\left(\frac{-1}{2}\right)x^{-3/2} + 0$$
$$= \frac{1}{12}x^{-1/2} - \frac{5}{2}x^{-3/2}$$

c. $f(x) = (\ln(x))(\tan(x))$

$$f'(x) = \frac{1}{x}\tan x + (\ln(x))\cdot\sec^2(x)$$
$$= \frac{\tan(x)}{x} + (\ln(x))\sec^2(x)$$

d. $f(x) = (x + 3^x + e^3)^5$

$$f'(x) = 5(x + 3^x + e^3)^4(1 + (\ln 3)3^x + 0)$$

$$= 5(1 + (\ln 3)3^x)(x + 3^x + e^3)^4$$

e. $f(x) = 4\sin^{-1}(4x)$

$$f'(x) = 4 \cdot \frac{1}{\sqrt{1-(4x)^2}} \cdot 4 = \frac{16}{\sqrt{1-16x^2}}$$

f. $f(x) = \dfrac{\cos(x)}{\sin(x)} = \cot(x)$.

or Quotient Rule

$$f'(x) = -\csc^2(x)$$

$$f'(x) = \frac{(\sin(x))(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)}$$

$$= \frac{-[\sin^2(x) + \cos^2(x)]}{\sin^2(x)}$$

g. $y=\left(x^{0.1}+1\right)^{-2/5}$

$$y' = -\frac{2}{5}\left(x^{0.1}+1\right)^{-7/5}\left(0.1x^{-0.9}\right)$$

h. $y=x^4e^{4x}+e^{-x}$

$$y' = 4x^3e^{4x} + x^4\cdot 4\cdot e^{4x} - e^{-x}$$
$$= 4x^3e^{4x} + 4x^4e^{4x} - e^{-x}$$

i. $f(x)=\dfrac{\sin(\pi/x)}{x^2+2} = \dfrac{\sin(\pi x^{-1})}{x^2+2}$

$$f'(x) = \frac{(x^2+2)\cos(\pi x^{-1})(-\pi x^{-2}) - \sin(\pi x^{-1})(2x)}{(x^2+2)^2}$$

$$= \frac{-\left[\pi x^{-2}(x^2+2)\cos(\pi x^{-1}) + 2x\sin(\pi x^{-1})\right]}{(x+2)^2}$$

OR PRODUCT RULE

$$= \left(\sin(\pi x^{-1})\right)\left(x^2+2\right)^{-1}$$

$$f'(x) = -\pi x^{-2}\cos(\pi x^{-1})(x^2+2)^{-1} + \sin(\pi x^{-1})(-1)(x^2+2)^{-2}(2x)$$

j. $f(x) = \dfrac{\cos(2)}{\sqrt[3]{\cos(x)}} = \cos(2)\left(\cos(x)\right)^{-1/3}$

$$f'(x) = \cos(2)\left(\frac{-1}{3}\right)(\cos(x))^{-4/3}(-\sin(x))$$

$$= \frac{\cos(2)}{3}\sin(x)(\cos(x))^{-4/3}$$

k. $f(x) = \ln\left(\dfrac{\sin^2(2x)}{2x+1}\right) = 2\ln(\sin(2x)) - \ln(2x+1)$

$$f'(x) = 2\left(\frac{2\cos(2x)}{\sin(2x)}\right) - \frac{2}{2x+1}$$

l. Find $\dfrac{dy}{dx}$ for $xe^y + 5(x^3 + y^3) = 0$. You must solve for $\dfrac{dy}{dx}$.

$$1 \cdot e^y + x \cdot e^y \frac{dy}{dx} + 15x^2 + 15y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(e^y + 15x^2)}{xe^y + 15y^2}$$