Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{\sqrt{x}}{4} + \frac{5}{\sqrt{x}} - \frac{6}{\sqrt{5}}$ $= \frac{1}{4}x^{\frac{1}{2}} + 5x^{-1/2} - \frac{6}{\sqrt{5}}$

$$f'(x) = \frac{1}{4} \cdot \frac{1}{2} x^{-1/2} + 5\left(\frac{-1}{2}\right)x^{-3/2} + 0$$

$$= \frac{1}{8}x^{-1/2} - \frac{5}{2}x^{-3/2}$$

b. $f(x) = (\ln(x))(\tan(x))$

$$f'(x) = \frac{1}{x}\tan x + (\ln(x)) \cdot \sec^2(x)$$

$$= \frac{\tan(x)}{x} + (\ln(x))\sec^2(x)$$

c. $y = 5\sec(5x)$

$$y' = 5\sec(5x)\tan(5x)(5)$$

$$= 25\sec(5x)\tan(5x)$$

d. $f(x) = \dfrac{\cos(x)}{\sin(x)} = \cot(x).$

or Quotient Rule

$f'(x) = -\csc^2(x)$

$f'(x) = \dfrac{(\sin(x))(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)}$

$= \dfrac{-[\sin^2(x) + \cos^2(x)]}{\sin^2(x)}$

e. $f(x) = 3\sin^{-1}(3x)$

$f'(x) = 3\left(\dfrac{1}{\sqrt{1-(3x)^2}}\right)(3)$

$= \dfrac{9}{\sqrt{1-9x^2}}$

f. $f(x) = (x + 5^x + e^5)^3$

$f'(x) = 3(x + 5^x + e^5)^2(1 + 5^x(\ln 5))$

g. $y = \left(x^{0.2}+1\right)^{-2/3}$

$$y' = -\frac{2}{3}\left(x^{0.2}+1\right)^{-5/3}\left(0.2\,x^{-0.8}\right)$$

h. $f(x) = \dfrac{\sin(\pi/x)}{x^4+4}$ $= \dfrac{\sin(\pi x^{-1})}{x^4+4}$

$$f'(x) = \frac{(x^4+4)\left(\cos(\pi x^{-1})(-\pi x^{-2})\right) - \sin(\pi x^{-1})(4x^3)}{(x^4+4)^2}$$

i. $y = e^{-x} + x^2 e^{2x}$

$$y' = -e^{-x} + 2x e^{2x} + 2x^2 e^{2x}$$

j. $f(x) = \ln\left(\frac{\sin^2(3x)}{2x+1}\right)$ $= 2\ln(\sin(3x)) - \ln(2x+1)$

$$f'(x) = 2\left(\frac{3\cos(3x)}{\sin(3x)}\right) - \frac{2}{2x+1}$$

k. $f(x) = \frac{\cos(2)}{\sqrt[3]{\cos(x)}}$ $= \cos(2)\left(\cos(x)\right)^{-1/3}$

$$f'(x) = \cos(2)\left(-\frac{1}{3}\right)\left(\cos(x)\right)^{-4/3}\left(-\sin(x)\right)$$

$$= \frac{\cos(2)}{3}\sin(x)\left(\cos(x)\right)^{-4/3}$$

l. Find $\frac{dy}{dx}$ for $xe^y + 5(x^2+y^2) = 0$. You must solve for $\frac{dy}{dx}$.

$$1 \cdot e^y + xe^y\frac{dy}{dx} + 10x + 10y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(e^y + 10x)}{xe^y + 10y}$$