Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = x^e + \frac{\pi}{2x} - \frac{4}{\pi^2}$ $= x^e + \frac{\pi}{2}x^{-1} - \frac{4}{\pi^2}$

$f'(x) = ex^{e-1} - \frac{\pi}{2}x^{-2}$

b. $y = x\sec(x)$

$y' = \sec(x) + x\sec(x)\tan(x)$

$= \sec(x)(1 + x\tan(x))$

c. $f(x) = \tan^3(4x)$

$f'(x) = 3(\tan(4x))^2(\sec^2(4x))(4)$

$= 12\tan^2(4x)\sec^2(4x)$

d. $f(x) = \tan^{-1}(x^2)$

$$f'(x) = \frac{2x}{1+x^4}$$

e. $f(x) = (\sin(x) + x^{-2.3})^5$

$$f'(x) = 5(\sin(x) + x^{-2.3})^4 (\cos(x) + (-2.3)x^{-3.3})$$

f. $f(x) = \dfrac{3}{\sin(x)} = 3\csc(x)$

$$f'(x) = -3\csc(x)\cot(x)$$

g. $y = e^{-x}\cos\left(\frac{x}{2}\right) = e^{-x}\cos(\frac{1}{2}x)$

$$y' = -e^{-x}\cos(\tfrac{1}{2}x) + e^{-x}\cdot(-\sin(\tfrac{1}{2}x))(\tfrac{1}{2})$$

h. $y = \ln\left(\sqrt{x^6 - x}\right) = \ln((x^6-x)^{\frac{1}{2}}) = \frac{1}{2}\ln(x^6-x) = \frac{1}{2}\ln(x(x^5-1))$

$$= \frac{1}{2}\left[\ln(x) + \ln(x^5-1)\right]$$

$$y' = \frac{1}{2}\left[\frac{1}{x} + \frac{5x^4}{x^5-1}\right]$$

i. $f(x) = \frac{e^x}{(x^2+2)^3} = \frac{e^x}{(x^2+2)^3}$

$$f'(x) = \frac{(x^2+2)^3(e^x) - e^x\cdot 3(x^2+2)^2 2x}{(x^2+2)^6}$$

j. $f(x) = \tan\left(x^2 - e^{4x}\right)$

$$f'(x) = \sec^2(x^2 - e^{4x})(2x - 4e^{4x})$$

k. $f(x) = \dfrac{x + 2\sin(x)}{\sin(8)} = \dfrac{1}{\sin(8)}\left[x + 2\sin(x)\right]$

$$f'(x) = \frac{1}{\sin(8)}(1 + 2\cos(x))$$

l. Find $\frac{dy}{dx}$ for $x^3 - y^4 = ye^{x}$. You must solve for $\frac{dy}{dx}$.

$$3x^2 - 4y^3\frac{dy}{dx} = \frac{dy}{dx}e^x + ye^x$$

$$3x^2 - ye^x = e^x\frac{dy}{dx} + 4y^3\frac{dy}{dx} = \frac{dy}{dx}(e^x + 4y^3)$$

$$\frac{dy}{dx} = \frac{3x^2 - ye^x}{e^x + 4y^3}$$