

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{5x}{3} + \frac{5}{3x^2} - \frac{\pi^2}{3} = \frac{5}{3}x + \frac{5}{3}x^{-2} - \frac{\pi^2}{3}$

$$f'(x) = \frac{5}{3} - \frac{10}{3}x^{-3}$$

b. $g(\theta) = e^{\theta} \tan(\theta)$

$$g'(\theta) = e^{\theta} \cdot \tan(\theta) + e^{\theta} \sec^2 \theta$$

c. $h(x) = \csc(x^2) = (\sin(x^2))^{-1}$ or



$$h'(x) = -\csc(x^2) \cot(x^2) (2x)$$

$$h'(x) = (-1) (\sin(x^2))^{-2} (\cos(x^2)) (2x)$$

d. $y = (x^{0.2} + 3)^{-2/5}$

$$y' = \left(-\frac{2}{5}\right) (x^{0.2} + 3)^{-7/5} (0.2x^{-0.8})$$

e. $f(t) = \sqrt{t^2 + \sin^2(t)} = \left(t^2 + (\sin t)^2\right)^{1/2}$

$$f'(t) = \frac{1}{2} \left(t^2 + (\sin(t))^2\right)^{-1/2} (2t + 2\sin(t)\cos(t))$$

f. $f(x) = x \arctan(x)$

$$\begin{aligned} f'(x) &= 1 \cdot \arctan(x) + x \left(\frac{1}{1+x^2}\right) \\ &= \arctan(x) + \frac{x}{1+x^2} \end{aligned}$$

$$\text{g. } f(x) = \frac{\sin(\pi/x)}{x^3+x} = \frac{\sin(\pi x^{-1})}{x^3+x}$$

$$f'(x) = \frac{(x^3+x)(\cos(\pi x^{-1})(-\pi x^{-2})) - (3x^2+1)(\sin(\frac{\pi}{x}))}{(x^3+x)^2}$$

$$\text{h. } y = \ln(5) + e^{x^2} + \sec(9x)$$

$$y' = 2xe^{x^2} + 9\sec(9x)\tan(9x)$$

$$\text{i. } g(x) = \frac{x^2+2}{8} + \ln(8+\cos(x))$$

$$g'(x) = \frac{2}{8}x + \frac{1}{8+\cos(x)} (-\sin(x))$$

$$j. j(x) = \frac{x \ln(x) - \sqrt{x}}{x} = \ln(x) - x^{-1/2}$$

$$j'(x) = \frac{1}{x} + \frac{1}{2} x^{-3/2}$$

k. $f(x) = \sqrt{2} \cos(1 + e^{-Kx})$ (Assume K is a fixed positive constant.)

$$f'(x) = -\sqrt{2} \sin(1 + e^{-Kx}) (e^{-Kx} (-K))$$

$$= \sqrt{2} K e^{-Kx} \sin(1 + e^{-Kx})$$

l. Find $\frac{dy}{dx}$ for $1 + xe^y = x^3 + y^2$

$$1 \cdot e^y + x e^y \frac{dy}{dx} = 3x^2 + 2y \frac{dy}{dx}$$

$$(x e^y - 2y) \left(\frac{dy}{dx} \right) = 3x^2 - e^y$$

$$\frac{dy}{dx} = \frac{3x^2 - e^y}{x e^y - 2y}$$