


Name: \_\_\_\_\_

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with**  $f'(x) =$ ,  $dy/dx =$ , or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a.  $g(\theta) = e^\theta \tan(\theta)$

$$g'(\theta) = e^\theta \cdot \tan(\theta) + e^\theta \sec^2 \theta$$

b.  $h(x) = \csc(x^3) = (\sin(x^3))^{-1}$  or


$$h'(x) = -\csc(x^3) \cot(x^3) (3x^2)$$

$$h'(x) = (-1)(\sin(x^3))^{-2} (\cos(x^3)) (3x^2)$$

c.  $f(x) = \frac{5x}{3} + \frac{5}{3x^2} - \frac{\pi^2}{3} = \frac{5}{3}x + \frac{5}{3}x^{-2} - \frac{\pi^2}{3}$

$$f'(x) = \frac{5}{3} - \frac{10}{3}x^{-3}$$

d.  $f(x) = x \arctan(x)$

$$f'(x) = 1 \cdot \arctan(x) + x \left( \frac{1}{1+x^2} \right)$$
$$= \arctan(x) + \frac{x}{1+x^2}$$

e.  $y = (x^{0.3} + 3)^{-1/5}$

$$y' = -\frac{1}{5} (x^{0.3} + 3)^{-6/5} (0.3x^{-0.7})$$

f.  $f(t) = \sqrt{t^2 + \sin^2(t)} = (t^2 + (\sin t)^2)^{1/2}$

$$f'(t) = \frac{1}{2} (t^2 + (\sin(t))^2)^{-1/2} (2t + 2\sin(t)\cos(t))$$

g.  $g(x) = \frac{x^2 + 2}{6} + \ln(8 + \cos(x))$

$$g'(x) = \frac{2}{6}x + \frac{1}{8 + \cos(x)} (-\sin(x))$$

h.  $f(x) = \frac{\sin(\pi/x)}{x^3 + x} = \frac{\sin(\pi x^{-1})}{x^3 + x}$

$$f'(x) = \frac{(x^3 + x)(\cos(\pi x^{-1}))(-\pi x^{-2}) - (3x^2 + 1)(\sin(\frac{\pi}{x}))}{(x^3 + x)^2}$$

i.  $y = \ln(9) + e^{x^2} + \sec(5x)$

$$y' = 2xe^{x^2} + 5 \sec(5x)\tan(5x)$$

j.  $f(x) = \sqrt{2} \cos(1 + e^{-Nx})$  (Assume  $N$  is a fixed positive constant.)

$$\begin{aligned} f'(x) &= -\sqrt{2} \sin(1 + e^{-Nx}) \left( e^{-Nx} (-N) \right) \\ &= \sqrt{2} N e^{-Nx} \sin(1 + e^{-Nx}) \end{aligned}$$

k.  $j(x) = \frac{x \ln(x) - \sqrt{x}}{x} = \ln(x) - x^{-1/2}$

$$j'(x) = \frac{1}{x} + \frac{1}{2} x^{-3/2}$$

l. Find  $\frac{dy}{dx}$  for  $1 + xe^y = x^3 + y^2$

$$1 \cdot e^y + x e^y \frac{dy}{dx} = 3x^2 + 2y \frac{dy}{dx}$$

$$(x e^y - 2y) \left( \frac{dy}{dx} \right) = 3x^2 - e^y$$

$$\frac{dy}{dx} = \frac{3x^2 - e^y}{x e^y - 2y}$$