

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(\theta) = \sin(3\theta^5 + 2\theta + 1)$

$$f'(\theta) = \cos(3\theta^5 + 2\theta + 1) (15\theta^4 + 2)$$

b. $p(x) = \frac{3}{\sqrt{2x}} + \left(\frac{x+8}{3}\right)^2 = \frac{3}{\sqrt{2}} x^{-1/2} + \left(\frac{x}{3} + \frac{8}{3}\right)^2$

$$p'(x) = \frac{3}{\sqrt{2}} \left(-\frac{1}{2}\right) x^{-3/2} + 2\left(\frac{x}{3} + \frac{8}{3}\right) \left(\frac{1}{3}\right)$$

c. $h(x) = \cot(x)$

$$h'(x) = -\csc^2(x)$$

d. $f(x) = \arcsin(x^{-2})$

$$f'(x) = \frac{1}{\sqrt{1 - (x^{-2})^2}} (-2x^{-3})$$

e. $f(t) = \sqrt{t + \cos^3(t)} = \left(t + (\cos(t))^3\right)^{\frac{1}{2}}$

$$f'(t) = \frac{1}{2} \left(t + (\cos(t))^3\right)^{-\frac{1}{2}} \left(1 + 3(\cos(t))^2(-\sin(t))\right)$$

$$= \frac{1 - 3\sin(t)\cos^2(t)}{2\sqrt{t^3 + \cos^3(t)}}$$

f. $f(x) = x^{5/3} \sec(x)$

$$f'(x) = \frac{5}{3} x^{2/3} \sec(x) + x^{5/3} \sec(x) \tan(x)$$

g. $f(x) = \frac{x}{x + \tan(x)}$

$$f'(x) = \frac{(x + \tan(x))(1) - x(1 + \sec^2 x)}{(x + \tan(x))^2}$$

h. $g(x) = (\sin(\ln(x)))^6$

$$g'(x) = 6(\sin(\ln(x)))^5 (\cos(\ln(x))) \left(\frac{1}{x}\right)$$

i. $f(x) = e^{5x}(2-x)$

$$f'(x) = 5e^{5x}(2-x) + e^{5x}(-1)$$

$$j. k(x) = \frac{x^2 \ln(x) + 5}{x} = x \ln(x) + 5x^{-1}$$

$$k'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 5x^{-2} = \ln(x) + 1 - \frac{5}{x^2}$$

k. $f(x) = x^p \ln(ax+3)$ (Assume p and a are fixed positive constants.)

$$f'(x) = px^{p-1} + \frac{a}{ax+3}$$

l. Find $\frac{dy}{dx}$ for $x+y+\pi = ye^x$

$$1 + \frac{dy}{dx} = 1 \cdot \frac{dy}{dx} \cdot e^x + ye^x$$

$$(1 - e^x) \frac{dy}{dx} = ye^x - 1$$

$$\frac{dy}{dx} = \frac{ye^x - 1}{1 - e^x}$$