

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{8x}{3} + \frac{8}{3x} + \ln(3) = \frac{8}{3}x + \frac{8}{3}x^{-1} + \ln(3)$

$$f'(x) = \frac{8}{3} - \frac{8}{3}x^{-2}$$

b. $f(t) = \cos(5 - \sqrt[3]{t}) = \cos(5 - t^{1/3})$

$$f'(t) = -\sin(5 - t^{1/3}) \left(-\frac{1}{3}t^{-2/3}\right)$$

c. $k(x) = \frac{\pi + \pi x}{1 + x^2}$

$$k'(x) = \frac{(1+x^2)(\pi) - (\pi+\pi x)(2x)}{(1+x^2)^2}$$

$$\text{d. } h(\theta) = \frac{1}{\sqrt{1-\theta^2}} = (1-\theta^2)^{-1/2}$$

$$h'(\theta) = -\frac{1}{2} (1-\theta^2)^{-3/2} (-2\theta)$$

$$\text{e. } g(x) = \arctan(x) + (\sin(x))^{-1}$$

$$g'(x) = \frac{1}{1+x^2} + (-1)(\sin(x))^{-2} (\cos(x))$$

$$\text{f. } f(x) = e^x \tan(x)$$

$$f'(x) = e^x \tan(x) + e^x \sec^2(x)$$

g. $j(z) = \cos(z + e^{9z})$

$$j'(z) = -\sin(z + e^{9z})(1 + 9e^{9z})$$

h. $g(x) = 7 \ln(x + x^2)$

$$g'(x) = \frac{7(1+2x)}{x+x^2}$$

i. $f(x) = (4-x) \sec(2x)$

$$f'(x) = (-1) \sec(2x) + (4-x) \sec(2x) \tan(2x) (2)$$

j. $f(x) = \ln(x + \sin(x^2))$

$$f'(x) = \frac{1}{x + \sin(x^2)} \cdot (1 + 2x \cos(x^2))$$

k. $f(x) = a^2x + e^{x+b}$ (Assume a and b are fixed positive constants.)

$$f'(x) = a^2 + e^{x+b}$$

l. Find $\frac{dy}{dx}$ for $(x \cdot y)^3 = 3x + 4y$

So $x^3 y^3 = 3x + 4y$.

$$3x^2 y^3 + x^3 \cdot 3y^2 \frac{dy}{dx} = 3 + 4 \frac{dy}{dx}$$

$$(3x^3 y^2 - 4) \frac{dy}{dx} = 3 - 3x^2 y^3$$

$$\frac{dy}{dx} = \frac{3 - 3x^2 y^3}{3x^3 y^2 - 4}$$