Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and put a "$+C$" in the correct place at least one time.

1. [12 points] Compute the integrals of the following functions.

a. $\int_{-1}^{1}(4x+2)\,dx$ $= 2x^2+2x\Big]_{-1}^{1}$

$= (2\cdot 1^2+2\cdot 1)-(2(-1)^2+2(-1)) = (4)-(0) = 4$

b. $\int_{0}^{1} x\sqrt{2x^2+2}\,dx$ $= \frac{1}{4}\int_2^4 u^{1/2}\,du = \frac{1}{4}\cdot\frac{2}{3}\,u^{3/2}\Big]_2^4 = \frac{1}{6}\left(4^{3/2}-2^{3/2}\right)$

let $u = 2x^2+2$ if $x=0$, $u=2$

$du = 4x\,dx$ $x=1$, $u=4$

$\frac{1}{4}du = x\,dx$

$= \frac{1}{6}(8-2\sqrt{2})$

$= \frac{4-\sqrt{2}}{3}$

c. $\int \left(\sin(5)+e^{-4x}\right)\,dx$

$= \sin(5)\,x - \frac{1}{4}e^{-4x} + C$

d. $\int (t - \sec^2(kt))\, dt$

$= \frac{1}{2}t^2 - \frac{1}{k}\tan(kt) + C$

e. $\int 7x^2 \cos(x^3 + 4)\, dx$ $= \frac{7}{3}\int \cos(u)\, du = \frac{7}{3}\sin(u) + C$

let $u = x^3 + 4$

$du = 3x^2 dx$

$\frac{1}{3}du = x^2 dx$

$= \frac{7}{3}\sin(x^3+4) + C$

f. $\int \frac{1}{\sqrt{1+25x^2}}\, dx$ $= \int \frac{dx}{\sqrt{1+(5x)^2}} = \frac{1}{5}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{5}\arcsin(u) + C$

Let $u = 5x$

$du = 5\, dx$

$\frac{1}{5}du = dx$

$= \frac{1}{5}\arcsin(5x) + C$

g. $\int 3\left(\frac{\pi x+4}{x}\right)\, dx$ $= 3\int(\pi + 4x^{-1})\,dx$

$= 3(\pi x + 4\ln|x|) + C$

h. $\int \frac{x+\sec(x)\tan(x)}{x^2+2\sec(x)}\, dx$ $= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+2\sec(x)| + C$

let $u = x^2 + 2\sec(x)$

$du = (2x + 2\sec(x)\tan(x))\,dx$

$\frac{1}{2}\,du = (x + \sec(x)\tan(x))\,dx$

i. $\int \frac{1}{x(\ln(x))^3}\, dx$ $= \int \frac{(\ln(x))^{-3}}{x}\,dx = \int u^{-3}\,du$

let $u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= -\frac{1}{2}u^{-2} + C$

$= -\frac{1}{2}(\ln(x))^{-2} + C$

j. $\int (x^{1.3} + \frac{e^x}{5} + \sin(x))\, dx$

$$= \frac{x^{2.3}}{2.3} + \frac{1}{5}e^x - \cos(x) + C$$

k. $\int \sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\, dx$ $= 2\sec\left(\frac{x}{2}\right) + C$

l. $\int 2x(3-4x)^8\, dx$ $= \int \left(\frac{3}{2} - \frac{1}{2}u\right)(u^8)\left(-\frac{1}{4}du\right) = -\frac{1}{8}\int (3-u)u^8\, du = -\frac{1}{8}\int (3u^8 - u^9)\, du$

let $u = 3-4x$

$du = -4\,dx$

$-\frac{1}{4}du = dx$

$4x = 3-u$

$2x = \frac{3}{2} - \frac{1}{2}u$

$$= -\frac{1}{8}\left(\frac{3}{9}u^9 - \frac{1}{10}u^{10}\right) + C$$

$$= -\frac{1}{8}\left(\frac{1}{3}(3-4x)^9 - \frac{1}{10}(3-4x)^{10}\right) + C$$

j. $\int (x^{1.3} + \frac{e^x}{5} + \sin(x))\, dx$

k. $\int \sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\, dx$

l. $\int 2x(3-4x)^8\, dx$