Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and do put a "$+C$" in the correct place at least one time.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int \left(\frac{x}{3}+\frac{4}{x}+\frac{4}{3}\right)\,dx \;=\; \boxed{\frac{1}{6}x^2 + 4\ln|x| + \frac{4}{3}x + C}$

b. $\int \cos(2x)\,dx \;=\; \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin v + C = \boxed{\frac{1}{2}\sin(2x)+C}$

Let $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

c. $\int_1^2 x(x+1)\,dx = \int_1^2 (x^2+x)\,dx = \left[\frac{x^3}{3}+\frac{x^2}{2}\right]_1^2 = \frac{2^3}{3}+\frac{2^2}{2}-\frac{1}{3}-\frac{1}{2} = \boxed{\frac{23}{6}}$

d. $\displaystyle\int \frac{1+\sec^2(2x)}{2x+\tan(2x)}\,dx$ $= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\text{Ln}|u| + C = \boxed{\frac{1}{2}\text{Ln}|2x+\tan(2x)| + C}$

Let $u = 2x + \tan(2x)$

$du = 2 + 2\sec^2(2x)$

$\frac{1}{2}du = 1 + \sec^2(2x)$

---

e. $\displaystyle\int \frac{5}{x(\ln x)^3}\,dx$ $= 5\int u^{-3}du = -\frac{5}{2}u^{-2} + C = \boxed{-\frac{5}{2}(\text{Ln}\,x)^{-2} + C}$

Let $u = \text{Ln}\,x$

$du = \frac{1}{x}dx$

f. $\displaystyle\int (4x^3+\sin(x)+e^x)\,dx$ $= \boxed{x^4 - \cos x + e^x + C}$

g. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \boxed{\sin^{-1}x + C}$

h. $\int \frac{x^3}{\sqrt{3-x^4}}\,dx = -\frac{1}{4}\int \frac{du}{\sqrt{u}} = -\frac{1}{4}(2\sqrt{u}) + C = \boxed{-\frac{1}{2}\sqrt{3-x^4} + C}$

Let $u = 3 - x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}du = x^3\,dx$

i. $\int (e^{-x} + \sec(x)\tan(x))\,dx = \boxed{-e^{-x} + \sec x + C}$

j. $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\tan^{-1}x)^2 + C}$

Let $u = \tan^{-1}x$

$du = \frac{1}{1+x^2}\,dx$

---

k. $\int_0^2 2x(x^2-2)^3\,dx = \int_{-2}^{2} u^3\,du = \boxed{0}$ since u^3 is an odd function

Let $u = x^2 - 2$

$du = 2x$

l. $\int \frac{x}{x-1}\,dx = \int \frac{u+1}{u}\,du = \int \left(1 + \frac{1}{u}\right)du = u + \ln|u| + C$

Let $u = x-1$ $= \boxed{x - 1 + \ln|x-1| + C}$

$du = dx$