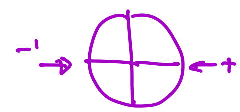
Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and do put a "$+C$" in the correct place at least one time.
- You must show sufficient work to justify your final expression; a correct answer for a non-trivial computation with no supporting work will be marked as incorrect.
- **Circle or box your final answer.**

1. [12 points] Compute the integrals of the following functions.

a. $\int_0^{\pi} 4x^3 + \sin x\, dx$

$= \frac{4x^4}{4} - \cos(x)\Big|_0^{\pi} = (\pi^4 - \cos(\pi)) - (0^4 - \cos(0))$



$= \pi^4 + 1 + 1$

$= \pi^4 + 2$

b. $\int (x^{1/3} + \frac{4}{x} + e^2)\, dx$

$= \frac{x^{4/3}}{4/3} + 4\ln|x| + xe^2 + C$

$= \frac{3x^{4/3}}{4} + 4\ln|x| + xe^2 + C$

c. $\int_0^1 t^2(4-t)\, dt$

$= \int_0^1 4t^2 - t^3\, dt$

$= \frac{4t^3}{3} - \frac{t^4}{4}\Big|_0^1 = \left(\frac{4}{3}(1)^3 - \frac{1^4}{4}\right) - \left(\frac{4}{3}(0)^3 - \frac{0^4}{4}\right)$

$= \frac{4}{3} - \frac{1}{4} = \frac{16}{12} - \frac{3}{12} = \frac{13}{12}$

d. $\int \sin t \cos t \, dt$ $= \int u \, du = \frac{u^2}{2} + c = \frac{(\sin t)^2}{2} + c$

$u = \sin t$

$du = \cos t \, dt$

e. $\int 3e^x (\sec(e^x))^2 \, dx$ $= \int 3e^x (\sec(u))^2 \frac{du}{e^x}$

$u = e^x$

$du = e^x dx$

$= 3\int (\sec(u))^2 \, du$

$= 3 \tan(u) + c$

$= 3 \tan(e^x) + c$

f. $\int \pi \left(\frac{7x-6}{2} \right) dx$

$= \frac{\pi}{2} \int 7x - 6 \, dx$

$= \frac{\pi}{2} \left(\frac{7x^2}{2} - 6x \right) + c = \frac{7\pi x^2}{4} - 3\pi x + c$

g. $\displaystyle\int \frac{1}{1+9x^2}\,dx$

$$= \int \frac{1}{1+(3x)^2}\,dx = \int \frac{1}{1+u^2}\cdot\frac{du}{3}$$

$u = 3x$

$\frac{du}{3} = dx$

$$= \frac{1}{3}\arctan(u) + C$$

$$= \frac{1}{3}\arctan(3x) + C$$

h. $\displaystyle\int \frac{x+\cos x}{2\sin x + x^2}\,dx = \int \frac{1}{u}\,\frac{du}{2}$

$u = 2\sin x + x^2$

$du = 2\cos(x) + 2x$

$= 2(x+\cos(x))$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|2\sin x + x^2| + C$$

i. $\displaystyle\int \frac{\ln x + 6}{x\ln x}\,dx = \int \frac{\ln x}{x\ln x} + \frac{6}{x\ln x}\,dx$

$$= \int \frac{1}{x}\,dx + \int \frac{6}{x\ln x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \ln|x| + \int \frac{6}{x\cdot u}(x\,du)$$

$$= \ln|x| + \int \frac{6}{u}\,du$$

$$= \ln|x| + 6\ln|\ln(x)| + C$$

j. $\int \sqrt{4x+5}\,dx \quad = \int \frac{1}{4}\sqrt{u}\;du = \int \frac{1}{4}\,u^{1/2}\,du$

$u = 4x+5$

$du = 4\,dx$

$= \frac{1}{4}\,\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{4}\cdot\frac{2}{3}\,u^{3/2} + C$

$= \frac{1}{6}(4x+5)^{3/2} + C$

k. $\int \sec(5x)\tan(5x)\,dx \quad = \int \sec(u)\tan(u)\cdot\frac{du}{5}$

$u = 5x$

$du = 5dx$

$\frac{du}{5} = dx$

$= \frac{1}{5}\sec(u) + C$

$= \frac{1}{5}\sec(5x) + C$

l. $\int x^3(x^4-7)^5\,dx \quad = \int u^5\cdot\frac{du}{4}$

$u = x^4 - 7$

$du = 4x^3\,dx$

$\frac{du}{4} = x^3\,dx$

$= \frac{1}{4}\int u^5\,du$

$= \frac{1}{4}\,\frac{u^6}{6} + C$

$= \frac{(x^4-7)^6}{24} + C$