Name: Solutions

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- You must show sufficient work to justify your final expression; a correct answer for a non-trivial computation with no supporting work will be marked as incorrect.
- Circle or box your final answer.

1. [12 points] Compute the integrals of the following functions.

a. $\int_1^4 \frac{x+1}{\sqrt{x}}\,dx = \int_1^4 \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}\,dx = \int_1^4 x^{1/2} + x^{-1/2}\,dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}\Big|_1^4 = \frac{2}{3}x^{3/2} + 2x^{1/2}\Big|_1^4 = \left(\frac{2}{3}(2)^3 + 2(2)\right) - \left(\frac{2}{3} + 2\right)$

$= \frac{16}{3} + 4 - \frac{2}{3} - 2 = \frac{14}{3} + 2 = \frac{14}{3} + \frac{6}{3} = \frac{20}{3} = 6\,2/3$

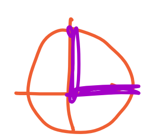
b. $\int_0^{1/2} (6 - \cos(\pi x))\,dx$

$u = \pi x \quad du = \pi\,dx$

$= \int_0^{1/2} 6\,dx - \int_0^{1/2} \cos(\pi x)\,dx = 6x\Big|_0^{1/2} - \frac{1}{\pi}\sin(\pi x)\Big|_0^{1/2}$

$= \left[6(1/2) - 0\right] - \frac{1}{\pi}\left(\sin(\pi/2) - \sin(0)\right) = 3 - \frac{1}{\pi}\left((1) - 0\right)$

$= 3 - \frac{1}{\pi}$



c. $\int (x+3)(5x+2)\,dx$

$= \int 5x^2 + 15x + 2x + 6\,dx = \int 5x^2 + 17x + 6\,dx$

$= \frac{5(x^3)}{3} + \frac{17x^2}{2} + 6x + C$

d. $\int xe^{9x^2}\,dx$ $= \int e^u \frac{du}{18} = \frac{1}{18}e^u + C$

$u = 9x^2$

$du = 18x\,dx$ $= \frac{1}{18}e^{9x^2} + C$

$\frac{du}{18} = x\,dx$

e. $\int \frac{\sin(x)-1}{\cos(x)+x}\,dx$ $= -\int \frac{1}{u}\,du = -\ln|u| + C$

$= -\ln|\cos(x)+x| + C$

$u = \cos(x) + x$

$du = -\sin(x) + 1$

$-du = \sin(x) - 1$

f. $\int \frac{e^x}{(12+e^x)^4}\,dx$ $= \int \frac{1}{u^4}\,du$

$u = 12 + e^x$ $= \int u^{-4}\,du$

$du = e^x\,dx$

$= \frac{u^{-3}}{-3} + C = -\frac{1}{3}(12+e^x)^{-3} + C$

g. $\int \sec(1-2x)\tan(1-2x)\,dx$ $= -\frac{1}{2}\int \sec(u)\tan(u)\,du$

$u = 1-2x$

$du = -2\,dx$

$\frac{du}{-2} = dx$

$= -\frac{1}{2}\sec(u) + C$

$= -\frac{1}{2}\sec(1-2x) + C$

h. $\int \frac{8}{1+x^2}\,dx$ $= 8\int \frac{1}{1+x^2}\,dx$

$= 8\arctan(x) + C$

i. $\int x(x+1)^{10}\,dx$ $= \int (u-1)u^{10}\,du$

$u = x+1$

$du = dx$

$x = u-1$

$= \int u^{11} - u^{10}\,du$

$= \frac{u^{12}}{12} - \frac{u^{11}}{11} + C$

$= \frac{(x+1)^{12}}{12} - \frac{(x+1)^{11}}{11} + C$

j. $\int \sqrt{2}\,(\sec(x))^2\,dx$

$= \sqrt{2} \int \sec^2(x)\,dx$

$= \sqrt{2}\,\tan(x) + C$

k. $\int \left(\frac{1}{x} + \frac{\ln(x)}{x}\right) dx$

$= \int \frac{1}{x}\,dx + \int \frac{\ln(x)}{x}\,dx = \int \frac{1}{x}\,dx + \int u\,du$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \ln|x| + \frac{(\ln|x|)^2}{2} + C$

alternately, $\int \frac{1}{x} + \frac{\ln x}{x}\,dx = \int \frac{1+\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2} + C =$

$u = 1 + \ln(x)$

$du = 1/x\,dx$

$\frac{1}{2}(1+\ln(x))^2 + C = \frac{1}{2}(1 + 2\ln(x) + (\ln(x))^2) + C$

$= \frac{1}{2} + \ln(x) + \frac{(\ln(x))^2}{2} + C$

pull this into the constant to match the other answer!

l. $\int \left(\sqrt[3]{x^5} + \sqrt[3]{4}\right) dx = \int x^{5/3} + \sqrt[3]{4}\,dx$

$= \frac{x^{8/3}}{8/3} + x\sqrt[3]{4} + C$