Name: ____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = e^t(5 - t^3)$

$f'(t) = e^t(5-t^3) - 3t^2 e^t$

b. $f(x) = \dfrac{\pi}{\sin x}$ $= \pi \csc(x)$

$f'(x) = -\pi \csc(x)\cot(x)$

(quotient rule works too)

c. $r(\theta) = \cot\left(2\sqrt{3} + \theta^5\right)$

$r'(\theta) = -5\theta^4 \csc^2(2\sqrt{3} + \theta^5)$

d. $f(r) = \dfrac{r^3 + \sqrt{r} - 8}{r}$ $= r^2 + r^{-1/2} - 8r^{-1}$

$f'(r) = 2r - \dfrac{r^{-3/2}}{2} + 8r^{-2}$

or $f'(r) = \dfrac{(3r^2 + \frac{1}{2\sqrt{r}})r - (r^3 + \sqrt{r} - 8)}{r^2}$

e. $G(x) = \left(\dfrac{x - \ln(3)}{2}\right)^4 - \sqrt{x+3}$

$G'(x) = 2\left(\dfrac{x - \ln(3)}{2}\right)^3 - \dfrac{1}{2\sqrt{x+3}}$

f. $g(z) = (7 - z)(z^3 + 6)$ $= 7z^3 + 42 - z^4 - 6z$

$g'(z) = 21z^2 - 4z^3 - 6$

(product rule works too)

g. $y(t) = \ln\left(4t + \sin(t^2)\right)$

$$y'(t) = \frac{4 + 2t\cos(t^2)}{4t + \sin(t^2)}$$

h. $y = x^{1/3} + e^{-x}\cos(x)$

$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3} - e^{-x}\cos(x) - e^{-x}\sin(x)$$

i. $f(x) = \dfrac{3\sec(ax)}{2x^3}$ (where a is a constant)

$$f'(x) = \frac{6ax^3\sec(ax)\tan(ax) - 18x^2\sec(ax)}{4x^6}$$

j. $f(y) = 5^y + \tan\left(y^{-2}\right)$

$$f'(y) = 5^y \ln(y) - 2y^{-3}\sec^2(y^{-2})$$

k. $g(x) = \arctan(e^{2x})$

$$g'(x) = \frac{2e^{2x}}{1+(e^{2x})^2}$$

l. Compute $\frac{dy}{dx}$ if $\ln y - x^2 y = 2x + 8$. You $\boxed{\text{must}}$ solve for $\frac{dy}{dx}$.

$$\frac{d}{dx}(\ln(y) - x^2 y) = \frac{d}{dx}(2x+8)$$

$$\frac{dy}{dx}\cdot\frac{1}{y} - \left(2xy + x^2\frac{dy}{dx}\right) = 2$$

$$\frac{dy}{dx}\left(\frac{1}{y} - x^2\right) = 2 + 2xy$$

$$\frac{dy}{dx} = \frac{2+2xy}{y^{-1}-x^2}$$