Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = (\pi - t^5)\sin(t)$

$f'(t) = -5t^4 \sin(t) + (\pi - t^5)\cos(t)$

b. $f(x) = \dfrac{\ln(x)}{e^x}$

$f'(x) = \dfrac{\left(\frac{1}{x} \cdot e^x - e^x \ln(x)\right)}{e^{2x}}$

c. $g(z) = (z + e^5)(e^2 - z^3)$

$g'(z) = (e^2 - z^3) - 3z^2(z + e^5)$

d. $r(\alpha) = \ln\left(\alpha^4 - \frac{\pi}{4}\right)$

$$r'(\alpha) = \frac{4\alpha^3}{\left(\alpha^4 - \frac{\pi}{4}\right)}$$

e. $g(y) = \frac{y^{5/2} + y^{-1} - \sqrt{5}}{\sqrt{y}} = y^2 + y^{-3/2} - \sqrt{5}\, y^{-1/2}$

$$g'(y) = 2y - \frac{3}{2} y^{-5/2} + \frac{\sqrt{5}}{2} y^{-3/2}$$

Or use quotient rule

$$g'(y) = \frac{\left(\frac{5}{2} y^{3/2} - y^{-2}\right)\sqrt{y} - \left(y^{5/2} + y^{-1} - \sqrt{5}\right)\left(\frac{1}{2\sqrt{y}}\right)}{y}$$

f. $F(x) = \sqrt{\frac{\ln(5) - x}{2}} - (x+3)^5$

$$F'(x) = -\frac{1}{4}\left(\frac{\ln(5) - x}{2}\right)^{-1/2} - 5(x+3)^4$$

g. $y(t) = \cos\left(5t^2 + \ln(t^3)\right)$

$$y'(t) = \left(10t + \frac{3t^2}{t^3}\right)\left(-\sin\left(5t^2 + \ln(t^3)\right)\right)$$

h. $f(x) = \dfrac{4x^a}{3\tan(ax)}$ (where a is a constant)

$$f'(x) = \frac{4ax^{a-1}(3\tan(ax)) - 4x^a(3a\sec^2(ax))}{(3\tan(ax))^2}$$

i. $f(y) = \log_5(y) + \cot\left(y^{-3}\right)$

$$f'(y) = \frac{1}{y\ln(5)} - 3y^{-4}(-\csc^2(y^{-3}))$$

j. $y = x^{2.7} + \sec^2(3x)\ln(-x)$

$$y' = 2.7x^{1.7} + 3\sec(3x)\tan(3x)(2\sec(3x))\ln(-x) + \sec^2(3x)\left(\frac{-1}{x}\right)$$

k. $g(\theta) = \cos(\theta^2)\arccos(\theta^2)$

$$g'(\theta) = 2\theta(-\sin(\theta^2))\arccos(\theta^2) + \cos(\theta^2)\left(-\frac{2\theta}{\sqrt{1-\theta^4}}\right)$$

l. Compute $\frac{dy}{dx}$ if $x^2 + y^2 = 16 + 3xy^2$. You $\boxed{\text{must}}$ solve for $\frac{dy}{dx}$.

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(16+3xy^2)$$

$$2x + 2y\frac{dy}{dx} = 3y^2 + 6xy\frac{dy}{dx}$$

$$(2y - 6xy)\frac{dy}{dx} = 3y^2 - 2x$$

$$\frac{dy}{dx} = \frac{3y^2 - 2x}{2y - 6xy}$$