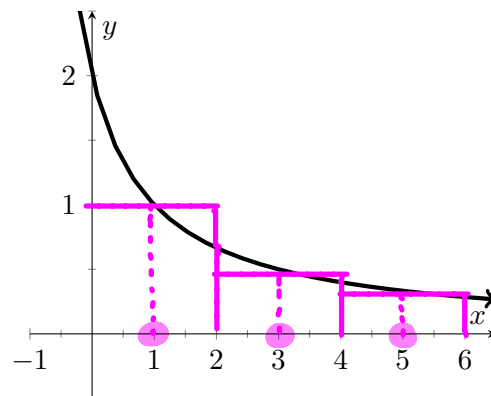
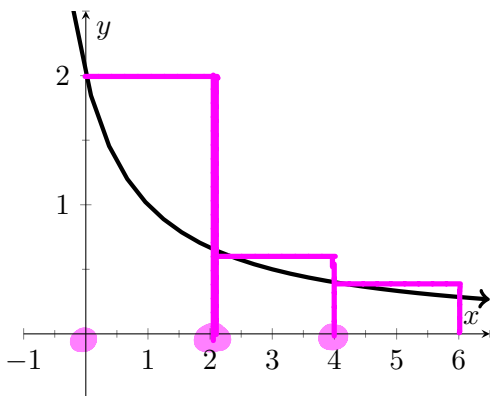


Name: \_\_\_\_\_ **Solutions** \_\_\_\_\_

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (9 pts.) Estimate the area under  $f(x) = \frac{2}{x+1}$  from  $x = 0$  to  $x = 6$  using three approximating rectangles and

- (a.) left endpoints. Sketch the rectangles on the graph below. (b.) midpoints as sample points. Sketch the rectangles on the graph below.



$$\begin{aligned}
 L_3 &= 2(f(0) + f(2) + f(4)) \\
 &= 2\left(\frac{2}{0+1} + \frac{2}{2+1} + \frac{2}{4+1}\right) \\
 &= 2\left(2 + \frac{2}{3} + \frac{2}{5}\right) = 2\left(\frac{30+10+6}{15}\right) \\
 &= \frac{2 \cdot 46}{15} = \frac{92}{15} = 6\frac{2}{15}
 \end{aligned}$$

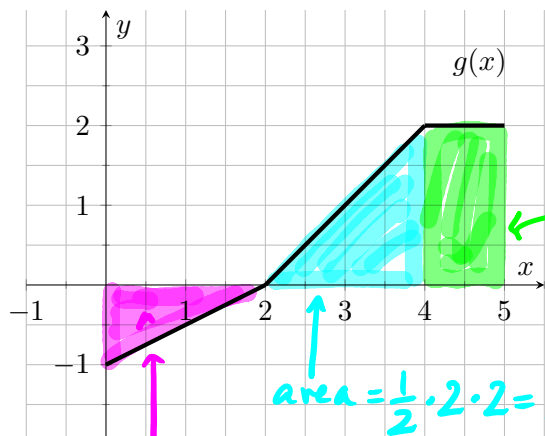
$$\begin{aligned}
 M_3 &= 2(f(1) + f(3) + f(5)) \\
 &= 2\left(\frac{2}{1+1} + \frac{2}{3+1} + \frac{2}{5+1}\right) \\
 &= 2\left(1 + \frac{1}{2} + \frac{1}{3}\right) = 2\left(\frac{6+3+2}{6}\right) \\
 &= \frac{11}{3} = 3\frac{2}{3}
 \end{aligned}$$

Exercise 2. (3 pts.) The speed of a skier increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a **lower estimate** for the distance she traveled during the first three seconds. Include units with your answer.

time (in seconds)	0	0.5	1	1.5	2	2.5	3
velocity (in feet/sec)	0	6	12	16	20	22	24

$$\text{distance} = \frac{1}{2}(0 + 6 + 12 + 16 + 20 + 22) = 3 + 6 + 8 + 10 + 11 = \underline{\underline{38 \text{ feet}}}$$

Exercise 3. (4 pts.) Use the graph of  $g(x)$  to evaluate the integral  $\int_0^5 g(x) dx$ .



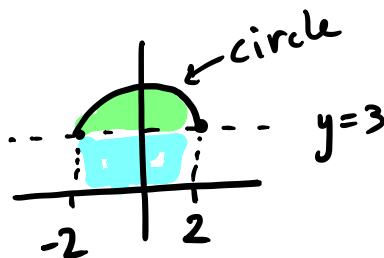
$$= \text{area above} - \text{area below} = 2 + 2 - 1 = 3$$

area =  $1 \cdot 2 = 2$

area =  $\frac{1}{2} \cdot 2 \cdot 2 = 2$

area =  $\frac{1}{2} \cdot 1 \cdot 2 = 1$

Exercise 4. (4 pts.) Evaluate the integral  $\int_{-2}^2 (\sqrt{4-x^2} + 3) dx$  by interpreting it in terms of areas.



↑  
top of circle

↑  
shifted 3 units up

$$\int_{-2}^2 (\sqrt{4-x^2} + 3) dx = \frac{1}{2} \pi \cdot 2^2 + 3 \cdot 4 = 2\pi + 12$$

Exercise 5. (5 pts.) Assume that  $\int_1^5 f(x) dx = 8$ . Use this fact and the properties of integrals to evaluate the integrals below.

(a.)  $\int_5^1 f(x) dx = -8$

↑  
limits are reversed

(b.)  $\int_1^5 (7 - 2\pi f(x)) dx$

$$= \int_1^5 7 - 2\pi \int_1^5 f(x) dx$$

$$= 7(5-1) - 2\pi \cdot 8$$

$$= 28 - 16\pi$$