Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #10, November 22nd**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (3 pts.) The speed of a skier increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance she traveled during the first three seconds. Include units with your answer.

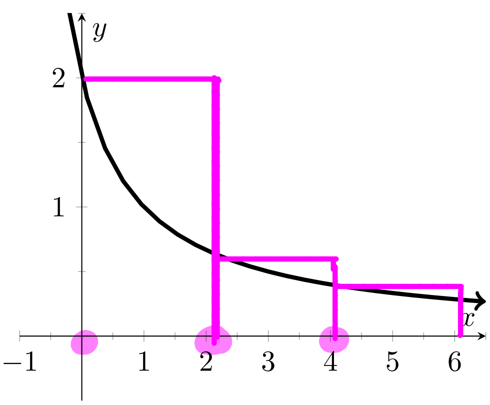
time (in seconds)	0	0.5	1	1.5	2	2.5	3
velocity (in feet/sec)	0	4	10	14	20	22	24

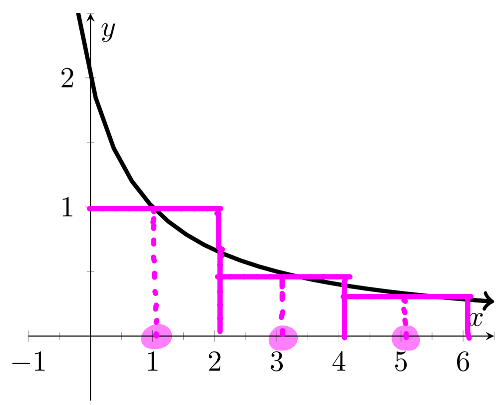
$$\text{distance} = \frac{1}{2}(0+4+10+14+20+22) = 2+5+7+10+11 = \underline{\underline{35 \text{ feet}}}$$

Exercise 2. (9 pts.) Estimate the area under $f(x) = \frac{2}{x+1}$ from $x = 0$ to $x = 6$ using three approximating rectangles and

(a.) left endpoints. Sketch the rectangles on the graph below.

(b.) midpoints as sample points. Sketch the rectangles on the graph below.

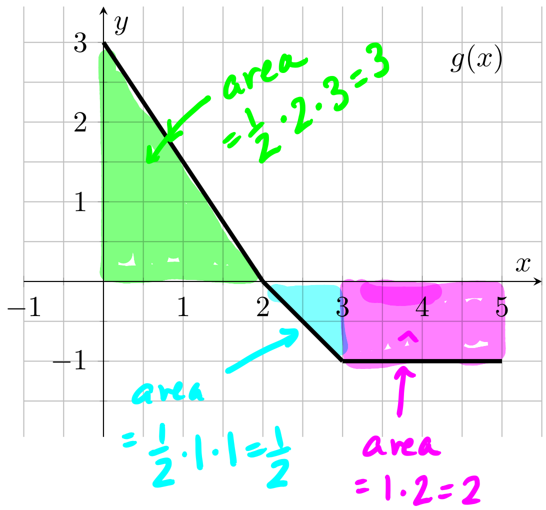




$$L_3 = 2(f(0)+f(2)+f(4))$$
$$= 2\left(\frac{2}{0+1}+\frac{2}{2+1}+\frac{2}{4+1}\right)$$
$$= 2\left(2+\frac{2}{3}+\frac{2}{5}\right) = 2\left(\frac{30+10+6}{15}\right)$$
$$= \frac{2\cdot 46}{15} = \frac{92}{15} = 6\tfrac{2}{15}$$

$$M_3 = 2(f(1)+f(3)+f(5))$$
$$= 2\left(\frac{2}{1+1}+\frac{2}{3+1}+\frac{2}{5+1}\right)$$
$$= 2\left(1+\frac{1}{2}+\frac{1}{3}\right) = 2\left(\frac{6+3+2}{6}\right)$$
$$= \frac{11}{3} = 3\tfrac{2}{3}$$

Exercise 3. (4 pts.) Use the graph of $g(x)$ to evaluate the integral $\int_0^5 g(x)\,dx$.



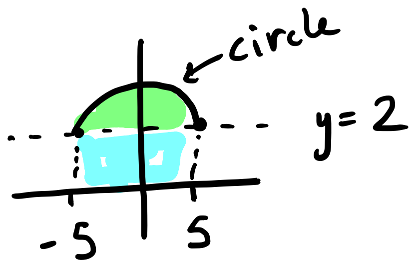
$$\int_0^5 g(x)\,dx = \text{area above} - \text{area below}$$

$$= 3 - \tfrac{1}{2} - 2 = \tfrac{1}{2}$$

Exercise 4. (4 pts.) Evaluate the integral $\int_{-5}^{5} (\sqrt{25-x^2}+2)\,dx$ by interpreting it in terms of areas.

top of circle

shifted 2 units up



$$\int_{-5}^{5}\left(\sqrt{25-x^2}+2\right)dx = \tfrac{1}{2}\pi\cdot 5^2 + 2\cdot 10 = \tfrac{25\pi}{2}+20$$

Exercise 5. (5 pts.) Assume that $\int_1^5 f(x)\,dx = 6$. Use this fact and the properties of integrals to evaluate the integrals below.

(a.) $\int_5^1 f(x)\,dx = -6$

limits are reversed

(b.) $\int_1^5 (7 - 2\pi f(x))dx$

$$= \int_1^5 7 - 2\pi\int_1^5 f(x)\,dx$$

$$= 7(5-1) - 2\pi\cdot 6$$

$$= 28 - 12\pi$$