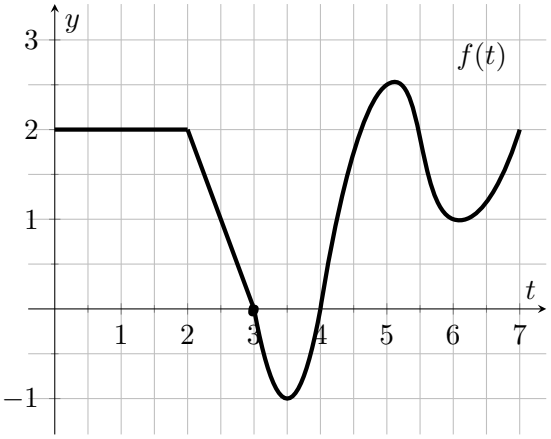
Circle your Instructor:
Faudree, Williams, Zirbes _____ / 25

Math 251 Fall 2017 **Quiz #11, November 29th**

Name: _____

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (3 pts.) Let $g(x) = \int_0^x f(t)dt$ where the graph of $y = f(t)$ is displayed below.



(a) Find $g(3) = \int_0^3 f(t)\,dt$

$= \boxed{5}$

(b) In the open interval $(0, 7)$, when does $g(x)$ have a maximum?

$\boxed{\text{at } x = 3}$

(c) When is $g(x)$ increasing? when $g'(x) = f(x)$ is positive or, on $\boxed{(0,3) \cup (4,7)}$

Exercise 2. (5 pts.) Find the derivative of the function.

(a) $g(x) = \int_x^1 \cos t\, dt = -\int_1^x \cos t\, dt$

$g'(x) = -\frac{d}{dx}\int_1^x \cos t\, dt$

$= \boxed{-\cos x}$

(b) $F(x) = \int_0^{x^3} \sqrt{1+t^4}\, dt$

$F'(x) = \sqrt{1+(x^3)^4} \cdot 3x^2$

$= \boxed{3x^2\sqrt{1+x^{12}}}$

Exercise 3. (3 pts.) What, if anything, is wrong with the following calculation?

$$\int_0^5 \frac{1}{x-2}dx = \ln|x-2||_0^5 = \ln 3 - \ln 2$$

The function $f(x) = \frac{1}{x-2}$ is discontinuous at $x = 2$, and thus this integral does not exist.

Exercise 4. (6 pts.) Evaluate the following integrals.

(a) $\int_0^{\pi/4} (\sec^2 t - 2e^t)\, dt$

$= (\tan t - 2e^t)\Big|_0^{\pi/4}$

$= \tan \pi/4 - 2e^{\pi/4} - (0-2)$

$= 1 - 2e^{\pi/4} + 2$

$= \boxed{3 - 2e^{\pi/4}}$

(b) $\int_0^{1/2} \frac{4}{\sqrt{1-x^2}}\, dx$

$= 4 \arcsin x \Big|_0^{1/2}$

$= 4 \arcsin(1/2) - 4 \arcsin 0$

$= 4(\pi/6)$

$= \boxed{2\pi/3}$

Exercise 5. (8 pts.) Evaluate the following integrals.

(a) $\int_0^1 (v^2+2)^2 dv$

$= \int_0^1 (v^4 + 4v^2 + 4) dv$

$= \left(\frac{1}{5}v^5 + \frac{4}{3}v^3 + 4v\right)\Big|_0^1$

$= \frac{1}{5} + \frac{4}{3} + 4 - 0$

$= \frac{3}{15} + \frac{20}{15} + \frac{60}{15}$

$= \boxed{\frac{83}{15}}$

(b) $\int_1^4 \frac{(4-t)}{\sqrt{t}} dt = \int_1^4 4t^{-1/2} - t^{1/2}\, dt$

$= \left(4 \cdot 2 t^{1/2} - \frac{2}{3} t^{3/2}\right)\Big|_1^4$

$= 8(2) - \frac{2}{3}(8) - (8 - 2/3)$

$= 16 - 16/3 - 8 + 2/3$

$= 8 - 14/3$

$= 24/3 - 14/3$

$= \boxed{10/3}$