

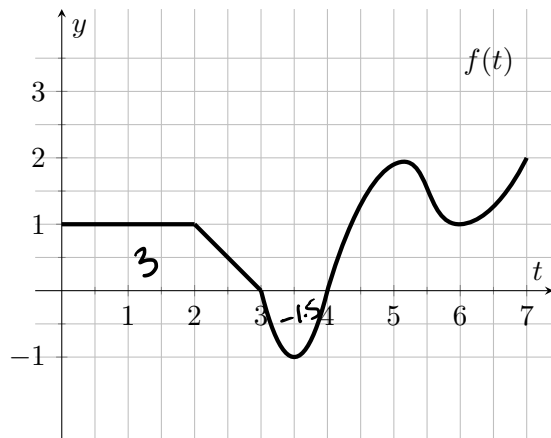
Math 251 Fall 2017

Quiz #11, November 29th

Name: \_\_\_\_\_

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (3 pts.) Let  $g(x) = \int_0^x f(t)dt$  where the graph of  $y = f(t)$  is displayed below.



(a) Find  $g(3) = \int_0^3 f(t)dt$

$$= 2 + 1/2 = \boxed{2.5}$$

(b) In the open interval  $(0, 7)$ , when does  $g(x)$  have a maximum?

$$\boxed{\text{at } x=3}$$

(c) When is  $g(x)$  increasing?

when  $g'(x) = f(x)$  is positive

$$\text{so on } \boxed{(0, 3) \cup (4, 7)}$$

Exercise 2. (5 pts.) Find the derivative of the function.

(a)  $g(x) = \int_x^2 \sin t dt = -\int_2^x \sin t dt$

$$\boxed{g'(x) = -\sin x}$$

(b)  $F(x) = \int_0^{x^2} \sqrt{2+t^3} dt$

$$F'(x) = 2x \sqrt{2+(x^2)^3}$$

$$\boxed{F'(x) = 2x \sqrt{2+x^6}}$$

Exercise 3. (3 pts.) What, if anything, is wrong with the following calculation?

$$\int_0^5 \frac{1}{x-3} dx = \ln|x-3| \Big|_0^5 = \ln 2 - \ln 3 = \ln\left(\frac{2}{3}\right)$$

The function  $f(x) = \frac{1}{x-3}$  is not continuous on  $[0, 5]$  and thus the integral does not exist.

Exercise 4. (6 pts.) Evaluate the following integrals.

$$\begin{aligned} \text{(a)} \quad & \int_0^{\pi/4} (2 \sec^2 t - e^t) dt \\ &= (2 \tan t - e^t) \Big|_0^{\pi/4} \\ &= 2 \tan \pi/4 - e^{\pi/4} - (0 - 1) \\ &= \boxed{3 - e^{\pi/4}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int_0^{1/2} \frac{2}{\sqrt{1-x^2}} dx = 2 \arcsin x \Big|_0^{1/2} \\ &= 2 \arcsin 1/2 - 2 \arcsin 0 \\ &= 2(\pi/6) \\ &= \boxed{\pi/3} \end{aligned}$$

Exercise 5. (8 pts.) Evaluate the following integrals.

$$\begin{aligned} \text{(a)} \quad & \int_0^1 (v^2 + 3)^2 dv \\ &= \int_0^1 (v^4 + 6v^2 + 9) dv \\ &= \left( \frac{1}{5} v^5 + 2v^3 + 9v \right) \Big|_0^1 \\ &= \frac{1}{5} + 2 + 9 \\ &= \frac{1}{5} + \frac{55}{5} \\ &= \boxed{\frac{56}{5}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int_1^9 \frac{(3-t)}{\sqrt{t}} dt = \int_1^9 (3t^{-1/2} - t^{1/2}) dt \\ &= \left( 6t^{1/2} - \frac{2}{3} t^{3/2} \right) \Big|_1^9 \\ &= 18 - \frac{2}{3}(27) - (6 - 2/3) \\ &= 18 - 18 - 6 + 2/3 \\ &= -18/3 + 2/3 \\ &= \boxed{-16/3} \end{aligned}$$