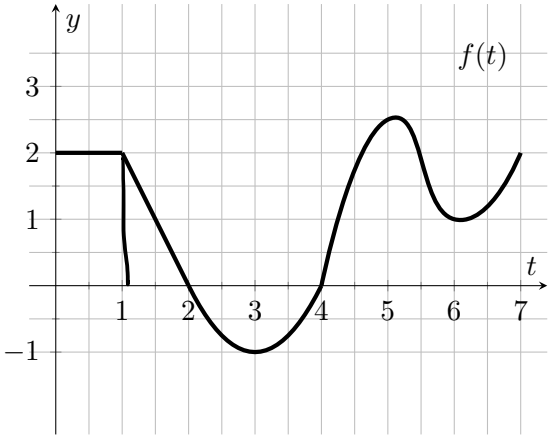
Circle your Instructor:
Faudree, Williams, Zirbes _____ / 25

Math 251 Fall 2017 **Quiz #11, November 29th**

Name: _____

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (3 pts.) Let $g(x) = \int_0^x f(t)dt$ where the graph of $y = f(t)$ is displayed below.



(a) Find $g(2) = \int_0^2 f(t)dt$

$= 2 + 1 = \boxed{3}$

(b) In the open interval $(0, 7)$, when does $g(x)$ have a maximum?

$\boxed{\text{at } x = 2}$

(c) When is $g(x)$ increasing?

when $g'(x) = f(x)$ is pos, so on $(0,2) \cup (4,7)$

Exercise 2. (5 pts.) Find the derivative of the function.

(a) $g(x) = \int_x^2 \sec^2 t\,dt = -\int_2^x \sec^2 t\,dt$

$\boxed{g'(x) = -\sec^2 x}$

(b) $F(x) = \int_0^{x^4} \sqrt{1+t^2}dt$

$F'(x) = 4x^3\sqrt{1+(x^4)^2}$

$\boxed{F'(x) = 4x^3\sqrt{1+x^8}}$

Exercise 3. (3 pts.) What, if anything, is wrong with the following calculation?

$$\int_0^6 \frac{1}{x-4}dx = \ln|x-4|\Big|_0^6 = \ln 2 - \ln 4 = \ln\left(\frac{2}{4}\right) = \ln\left(\frac{1}{2}\right)$$

As the function $f(x) = \frac{1}{x-4}$ is not continuous on $[0,6]$ this integral does not exist.

Exercise 4. (6 pts.) Evaluate the following integrals.

(a) $\int_0^{\pi/4} (2\sec^2 t - e^t)dt$

$= (2\tan t - e^t)\Big|_0^{\pi/4}$

$= 2 - e^{\pi/4} - (0 - 1)$

$= \boxed{3 - e^{\pi/4}}$

(b) $\int_0^{1/2} \frac{3}{\sqrt{1-x^2}}dx$

$= 3\arcsin x \Big|_0^{1/2}$

$= 3\arcsin(1/2) - 3\arcsin 0$

$= 3(\pi/6)$

$= \boxed{\pi/2}$

Exercise 5. (8 pts.) Evaluate the following integrals.

(a) $\int_0^1 (v^2+1)^2 dv$

$= \int_0^1 (v^4 + 2v^2 + 1)\,dv$

$= \left(\frac{1}{5}v^5 + \frac{2}{3}v^3 + v\right)\Big|_0^1$

$= \frac{1}{5} + \frac{2}{3} + 1$

$= \frac{3}{15} + \frac{10}{15} + \frac{15}{15}$

$= \boxed{\frac{28}{15}}$

(b) $\int_1^4 \frac{(2-t)}{\sqrt{t}}dt = \int_1^4 (2t^{-1/2} - t^{1/2})\,dt$

$= \left(4t^{1/2} - \frac{2}{3}t^{3/2}\right)\Big|_1^4$

$= 8 - \frac{2}{3}(8) - (4 - 2/3)$

$= 4 - 16/3 + 2/3$

$= 12/3 - 14/3$

$= \boxed{-2/3}$