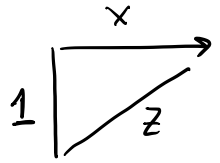
Circle your Instructor:
Faudree, Williams, Zirbes _____ / 25

Math 251 Fall 2017 **Quiz #7 (Sample), October 25th**

Name: _____

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (5 pts.) A plane flying horizontally at an altitude of 1 mile and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the *distance* from the plane to the station is increasing when it is 2 miles away from the station. Give your answer with proper units.



know: $\frac{dx}{dt} = 500$

want $\frac{dz}{dt}$ when $z = 2$

$x^2 + 1^2 = 2^2$

$x^2 + 1 = 4$

$x = \sqrt{3}$

$x^2 + 1 = z^2$

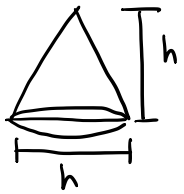
$2x \frac{dx}{dt} + 0 = 2z \frac{dz}{dt}$

$x \frac{dx}{dt} = z \frac{dz}{dt}$

$\sqrt{3}(500) = 2 \frac{dz}{dt}$

$\frac{dz}{dt} = 250\sqrt{3}$ mph

Exercise 2. (5 pts.) Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast ist he height of the pile increasing when the pile is 10 feet high? Give your answer with proper units. ($V = \frac{1}{3}\pi r^2 h$.)



note $r = \frac{1}{2}h$

Note $\frac{dV}{dt} = 30\ ft^3/min$

want dh/dt when $h = 10$

$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{1}{4} h^2 h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$30 = \frac{\pi}{4} \cdot 10^2 \frac{dh}{dt}$

$120 = 100\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{120}{100\pi}$

$= \frac{12}{10\pi}$

$= \frac{6}{5\pi} \frac{ft}{min}$

Exercise 3. (5 pts.) Strontium-90 has a half-life of 28 days.

(a) (2 points) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t days.

$m(t) = 50(1/2)^{t/28}$

(b) (1 point) Give an expression that finds the mass remaning after 50 days.

$m(50) = 50(1/2)^{50/28} = 50(1/2)^{25/14}$

(c) (2 points) Give an expression that will give the time for the sample to decay to a mass of 2 mg.

$2 = 50(1/2)^{t/28}$

$1/25 = (1/2)^{t/28}$

$\ln(1/25) = \frac{t}{28}\ln(1/2)$

$28\ln(1/25) = t\ln(1/2)$

$t = \frac{28\ln(1/25)}{\ln(1/2)}$

Exercise 4. (4 pts.) Find the linearization of $f(x) = e^x \cos x$ at $a = 0$.

$f'(x) = e^x\cos x - e^x\sin x$

$f'(0) = 1$

$L(x) = f'(a)(x-a) + f(a)$

$= 1(x-0) + 1$

$= x + 1$

Exercise 5. (6 pts.) The edge of a cube was found to be 30 cm with a possible error of 0.1 cm. Use differentials to estimate the maximum possible error in computing the

(a) volume of the cube.

$V = x^3$

$dV = 3x^2\,dx$

$dV = 3(30)^2(0.1)$

$= 3(900)(0.1)$

$= 3(90)$

$= 270\ cm^3$

(b) surface area of the cube.

$A = 6x^2$

$dA = 12x\,dx$

$dA = 12(30)(0.1)$

$= 12(3)$

$= 36\ cm^2$