Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #6, October 18th**

Name: Solution

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (4 pts.) Find $\frac{dy}{dx}$ by implicit differentiation for $\sin y = x^2 - y$.

$$\frac{d}{dx}\sin y = \frac{d}{dx}(x^2 - y)$$

$$\cos y \cdot y' = 2x - y'$$

$$y'(1+\cos y) = 2x$$

$$\boxed{y' = \frac{2x}{1+\cos y}}$$

Exercise 2. (6 pts.) Find the derivatives of the following functions.

(a) $f(x) = x\arcsin(2x)$ (b) $g(x) = \arctan(\sqrt{x})$

chain rule!

$f'(x) = u'v + uv'$

$= 1\cdot\arcsin(2x) + x\cdot\frac{1}{\sqrt{1-4x^2}}\cdot 2$

$= \arcsin(2x) + \frac{2x}{\sqrt{1-4x^2}}$

$g'(x) = \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}+2x\sqrt{x}}$

chain rule!

Exercise 3. (3 pts.) Find the derivative of the function $g(x) = \sqrt{\ln x}$.

$$g'(x) = \frac{1}{2\sqrt{\ln x}}\cdot\frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$$

Exercise 4. (4 pts.) Use logarithmic differentiation to find the derivative of the function

$$y = (\cos x)^{2x}.$$

$\ln(y) = \ln\left((\cos x)^{2x}\right)$

$\ln(y) = 2x \cdot \ln(\cos(x))$

$\frac{1}{y} \cdot y' = 2\ln(\cos(x)) + 2x \cdot \frac{1}{\cos(x)} \cdot (-\sin(x))$

$y' = \left[2\ln(\cos(x)) - 2x\tan(x)\right] \cdot (\cos x)^{2x}$

Exercise 5. (8 pts.) The position function of a particle is given by $s = \frac{1}{3}t^3 - 4t^2 + 12t$ where t is measured in seconds and s in meters. Further, assume the first and second derivatives are $s'(t) = t^2 - 8t + 12$ and $s''(t) = 2t - 8$.

a.) What is the velocity function of the particle?

$s'(t) = t^2 - 8t + 12$

b.) What is the acceleration function of the particle?

$s''(t) = 2t - 8$

c.) When is the particle at rest?

when $s'(t) = 0$, i.e., when $t^2 - 8t + 12 = (t-6)(t-2) = 0$

or $t = 2, 6$.

d.) When is the particle moving to the right?

when $s'(t) > 0$, i.e., when t in $(-\infty, 2) \cup (6, \infty)$

e.) At time $t = 3$, is the particle speeding up or slowing down? Explain your answer.

$s'(3) < 0$ and $s''(3) = 2 \cdot 3 - 8 = -2 < 0$

so velocity and acceleration in same direction.

thus speeding up.