Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #6, October 18th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (4 pts.) Find $\frac{dy}{dx}$ by implicit differentiation for $\cos y = x^2 - y$.

$\frac{d}{dx}\cos y = \frac{d}{dx}\left[x^2 - y\right]$

$-\sin(y)\cdot y' = 2x - y'$

$y'(1-\sin(y)) = 2x$

$y' = \frac{2x}{1-\sin y}$

Exercise 2. (6 pts.) Find the derivatives of the following functions.

(a) $f(x) = x\arccos(2x)$ (u v)

$f'(x) = u'v + uv'$

$f'(x) = 1\cdot\arccos(2x) + x\cdot\frac{-1}{\sqrt{1-4x^2}}\cdot 2$

$= \arccos(2x) - \frac{2x}{\sqrt{1-4x^2}}$

(b) $g(x) = \arctan(\sqrt{x})$ ← chain rule

$g'(x) = \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}+2x\sqrt{x}}$

Exercise 3. (3 pts.) Find the derivative of the function $g(x) = \sqrt{\ln x}$. ← chain rule!

$g'(x) = \frac{1}{2\sqrt{\ln(x)}}\cdot\frac{1}{x} = \frac{1}{2x\sqrt{\ln(x)}}$

Exercise 4. (4 pts.) Use logarithmic differentiation to find the derivative of the function

$$y = (\sin x)^{2x}.$$

$\ln(y) = \ln([\sin(x)]^{2x})$

$\ln(y) = 2x \ln(\sin x)$

$\frac{1}{y} \cdot y' = 2\ln(\sin x) + 2x \cdot \frac{1}{\sin x} \cdot \cos(x)$

$y' = [2\ln(\sin x) + 2x \cos x](\sin(x))^{2x}$

Exercise 5. (8 pts.) The position function of a particle is given by $s = \frac{1}{3}t^3 - 3t^2 + 5t$ where t is measured in seconds and s in meters. Further, assume the first and second derivatives are $s'(t) = t^2 - 6t + 5$ and $s''(t) = 2t - 6$.

a.) What is the velocity function of the particle?

$s'(t) = t^2 - 6t + 5$

b.) What is the acceleration function of the particle?

$s''(t) = 2t - 6$

c.) When is the particle at rest?

when $s'(t) = 0$, i.e., when $t^2 - 6t + 5 = (t-5)(t-1) = 0$

or when $t = 1, 5$.

d.) When is the particle moving to the right?

When $s'(t) > 0$, so when t in $(-\infty, 1) \cup (5, \infty)$

e.) At time $t = 2$, is the particle speeding up or slowing down? Explain your answer.

$s'(2) < 0$ and $s''(2) = 2 \cdot 2 - 6 < 0$

So speed and acceleration in same direction.

Thus the particle is speeding up.