

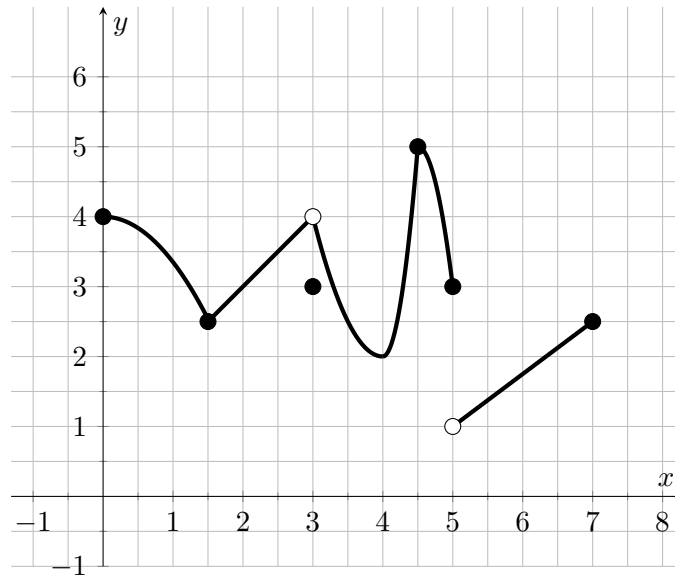
Math 251 Fall 2017

Quiz #8, November 1st

Name: Solutions

There are 23 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (8 pts.) Consider the graph of the function  $f$  given below.



a) State the absolute maximum of the function  $f$  on the interval  $[0, 6]$  and give its location or explain why it doesn't exist.

5 at  $x = 4.5$

b) State the absolute minimum of the function  $f$  on the interval  $[0, 6]$  and give its location or explain why it doesn't exist.

None. Graph approaches 1 as  $x=5$ , but doesn't reach it.

c) Identify any other local maxima of the function  $f$  and their locations.

4 at  $x=0$ , 2.5 at  $x=7$

d) Identify any other local minima of the function  $f$  and their locations.

2.5 at  $x=1.5$ , 3 at  $x=3$ , 2 at  $x=4$

Exercise 2. (5 pts.) Find the absolute maximum and absolute minimum of the function

$$f(x) = -x^3 - 3x^2 + 9x$$

on the interval  $[0, 3]$ .

$$\begin{aligned} f'(x) &= -3x^2 - 6x + 9 = -3(x^2 + 2x - 3) \\ &= -3(x+3)(x-1) \end{aligned}$$

so critical point at  $x=1$  in  $[0, 3]$

$$f(0) = 0$$

$$f(1) = -1 - 3 + 9 = 5$$

$$f(3) = -27 - 27 + 27 = -27$$

so 5 is absolute maximum.

-27 is absolute minimum.

Exercise 3. (5 pts.) Find the critical numbers of the function  $F(x) = x^{3/5}(x-3)$ .

$$F(x) = x^{8/5} - 3x^{3/5}$$

$$F'(x) = \frac{8}{5}x^{3/5} - \frac{9}{5}x^{-2/5} = \frac{x^{-2/5}}{5}(8x-9)$$

so critical numbers are  $x=0$  and

$$x = \frac{9}{8}.$$

Exercise 4. (5 pts.) Consider the function  $f(x) = 3x^2 - 2x + 1$  on the interval  $[0, 2]$ .

- a) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the interval  $[0, 2]$ . Justify your answer in words.

$f(x)$  is a polynomial, so it is continuous and differentiable on all of  $[0, 2]$ .

- b) Find all numbers  $c$  in the interval  $[0, 2]$  that satisfy the conclusion of the Mean Value Theorem.

$$m = \frac{f(2) - f(0)}{2} = \frac{12 - 4 + 1 - 1}{2} = \frac{8}{2} = 4$$

$$f'(x) = 6x - 2$$

$$6x - 2 = 4$$

$$6x = 6$$

$$x = 1$$

$$\boxed{\text{So } c = 1}$$