Circle your Instructor:
Faudree, Williams, Zirbes ________ / 25

Math 251 Fall 2017 **Quiz #9, November 8th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (12 pts.) Evaluate the limits below. Use L'Hospital's Rule where appropriate. **Indicate when you are using L'Hospital's Rule and state explicitly the indeterminate form.**

(a.) $\lim_{\theta \to \pi/2} \frac{\cos\theta}{\sin(6\theta)} \overset{H}{=} \lim_{\theta \to \frac{\pi}{2}} \frac{-\sin\theta}{6\cos(6\theta)} = \frac{-\sin(\pi/2)}{6\cos(3\pi)} = \frac{-1}{-6} = \frac{1}{6}$

$\cos(\pi/2) = 0$

$\sin(6 \cdot \frac{\pi}{2}) = \sin 3\pi = 0$

form $\frac{0}{0}$

(b.) $\lim_{x \to \infty} x^4 e^{-x^3} = \lim_{x \to \infty} \frac{x^4}{e^{+x^3}} \overset{H}{=} \lim_{x \to \infty} \frac{4x^3}{-3x^2 e^{x^3}} = \lim_{x \to \infty} \frac{-4x}{3e^{x^3}}$

form $\infty \cdot 0$ form $\frac{0}{0}$ form $\frac{0}{0}$

$\overset{H}{=} \lim_{x \to \infty} \frac{-4}{9x^2 e^{x^3}} = 0$

form 1^∞.

(c.) $\lim_{x \to 0} (1 - 5x)^{1/x} = \boxed{e^{-5}}$

retranslate

Let $y = (1-5x)^{1/x}$.

So $\ln y = \frac{1}{x}\ln(1-5x)$.

Now $\lim_{x \to 0} \frac{\ln(1-5x)}{x} \overset{H}{=} \lim_{x \to 0} \frac{\frac{-5}{1-5x}}{1} = -5$

form $\frac{0}{0}$

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Exercise 2. (10 pts.) Use the information below to answer questions about the function $f(x)$. Make sure you answer the question! If something doesn't exist, you must explicitly state this.

$$f(x) = \frac{1}{x^2+6}, \quad \underline{f'(x) = \frac{-2x}{(x^2+6)^2}}, \quad \underline{f''(x) = \frac{6(x^2-2)}{(x^2+6)^3}}.$$

(a.) Find the interval(s) where the function is increasing and the interval(s) where the function is decreasing.

$f' = 0$ when $x = 0$

f' never undefined

$+ + + + 0 - - - -$ ← Sign of f'

$-1 \quad 0 \quad 1$

$f'(1) = \frac{-}{+} = -$

$f'(-1) = \frac{+}{+} = +$

Answer:

f is increasing on $(-\infty, 0)$

and

decreasing on $(0, \infty)$

(b.) Find the local maximum and minimum values of f and where they occur.

$+ + + + 0 - - - -$ ← Sign of f'

$-1 \quad 0 \quad 1$

↑ local max here

answer:

$f(0) = \frac{1}{6}$ is the local maximum.

f has no local minimum.

(c.) Find the intervals of concavity and any inflection points.

$f''(x) = 0$ when $x^2 - 2 = 0$

or $x = \pm\sqrt{2}$

f'' is never undefined.

$+ + + + \; 0 - - - - - \; 0 + + +$ ← Sign of f''

$-5 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 5$

$f''(-5) = \frac{+ \cdot +}{+} = +$

$f''(0) = \frac{+ \cdot -}{+} = -$

$f''(5) = \frac{+ \cdot +}{+} = +$

$f(\sqrt{2}) = \frac{1}{(\sqrt{2})^2 + 6} = \frac{1}{8}$

$f(-\sqrt{2}) = \frac{1}{8}$

Answer:

f is ccup on $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$;

ccdown on $(-\sqrt{2}, \sqrt{2})$.

inflection points at $(\sqrt{2}, 1/8)$ and $(-\sqrt{2}, 1/8)$.

Exercise 3. (2 pts.) Given $f(x) = 2 + xe^{-x}$, $f'(x) = -(x-1)e^{-x}$, and $f''(x) = (x-2)e^{-x}$, use the Second Derivative Test to identify the local minimum and maximum values of f or explain why the test is inconclusive.

$f'(x) = 0$ when $x = 1$.

$f''(1) = -1 \cdot e^{-1} < 0$.

So f is ccdown at $x = 1$.

So f has a local maximum at $x = 1$. The maximum value is $f(1) = 2 + e^{-1}$.