

Math 251 Fall 2017

Quiz #9, November 8th

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. Please show all of your work! If you have any questions, please raise your hand.

Exercise 1. (12 pts.) Evaluate the limits below. Use L'Hospital's Rule where appropriate. Indicate when you are using L'Hospital's Rule and state explicitly the indeterminate form.

$$(a.) \lim_{\theta \rightarrow \pi/2} \frac{\cos \theta}{\sin(6\theta)} \stackrel{(H)}{=} \lim_{\theta \rightarrow \pi/2} \frac{-\sin \theta}{6 \cos(6\theta)} = \frac{-\sin(\pi/2)}{6 \cos(3\pi)} = \frac{-1}{-6} = \frac{1}{6}$$

$$\cos(\pi/2) = 0$$

$$\sin(6 \cdot \frac{\pi}{2}) = \sin 3\pi = 0$$

form  $\frac{0}{0}$

$$(b.) \lim_{x \rightarrow \infty} x^4 e^{-x^3} = \lim_{x \rightarrow \infty} \frac{x^4}{e^{x^3}} \stackrel{(H)}{=} \lim_{x \rightarrow \infty} \frac{4x^3}{-3x^2 e^{x^3}} = \lim_{x \rightarrow \infty} \frac{-4x}{3e^{x^3}}$$

$\uparrow$  form  $\infty \cdot 0$                        $\uparrow$  form  $\frac{0}{0}$                                        $\uparrow$  form  $\frac{0}{\infty}$

$$\stackrel{(H)}{=} \lim_{x \rightarrow \infty} \frac{-4}{9x^2 e^{x^3}} = 0$$

$$(c.) \lim_{x \rightarrow 0} (1 - 5x)^{1/x} = \boxed{e^{-5}}$$

$\swarrow$  form  $1^\infty$ .

Let  $y = (1 - 5x)^{1/x}$ .

So  $\ln y = \frac{1}{x} \ln(1 - 5x)$ .

$$\text{Now } \lim_{x \rightarrow 0} \frac{\ln(1 - 5x)}{x} \stackrel{(H)}{=} \lim_{x \rightarrow 0} \frac{\frac{-5}{1 - 5x}}{1} = -5$$

$\uparrow$  form  $\frac{0}{0}$

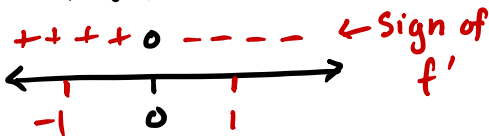
$\swarrow$  retranslate

Exercise 2. (10 pts.) Use the information below to answer questions about the function  $f(x)$ . Make sure you answer the question! If something doesn't exist, you must explicitly state this.

$$f(x) = \frac{1}{x^2 + 6}, \quad f'(x) = \frac{-2x}{(x^2 + 6)^2}, \quad f''(x) = \frac{6(x^2 - 2)}{(x^2 + 6)^3}.$$

(a.) Find the interval(s) where the function is increasing and the interval(s) where the function is decreasing.

$f' = 0$  when  $x = 0$   
 $f'$  never undefined



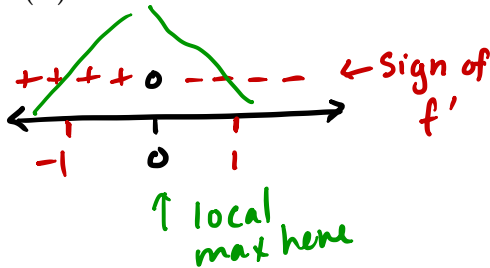
$$f'(1) = \frac{-}{+} = -$$

$$f'(-1) = \frac{+}{+} = +$$

Answer:

$f$  is increasing on  $(-\infty, 0)$   
and  
decreasing on  $(0, \infty)$

(b.) Find the local maximum and minimum values of  $f$  and where they occur.



Answer:

$f(0) = \frac{1}{6}$  is the local maximum.

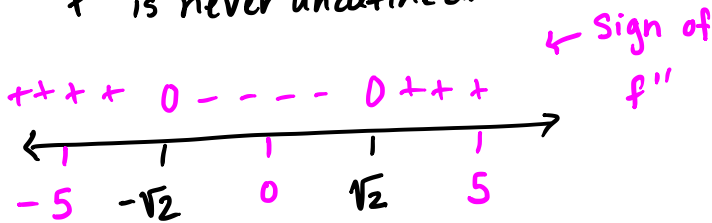
$f$  has no local minimum.

(c.) Find the intervals of concavity and any inflection points.

$$f''(x) = 0 \text{ when } x^2 - 2 = 0$$

$$\text{or } x = \pm\sqrt{2}$$

$f''$  is never undefined.



$$f''(-5) = \frac{+ \cdot +}{+} = +$$

$$f''(0) = \frac{+ \cdot -}{+} = -$$

$$f''(5) = \frac{+ \cdot +}{+} = +$$

$$f(\sqrt{2}) = \frac{1}{(\sqrt{2})^2 + 6} = \frac{1}{8}$$

$$f(-\sqrt{2}) = \frac{1}{8}$$

Answer:

$f$  is conc up on  $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ ;

cc down on  $(-\sqrt{2}, \sqrt{2})$ .

Inflection points at

$(\sqrt{2}, \frac{1}{8})$  and

$(-\sqrt{2}, \frac{1}{8})$ .

Exercise 3. (2 pts.) Given  $f(x) = 2 + xe^{-x}$ ,  $f'(x) = -(x - 1)e^{-x}$ , and  $f''(x) = (x - 2)e^{-x}$ , use the Second Derivative Test to identify the local minimum and maximum values of  $f$  or explain why the test is inconclusive.

$$f'(x) = 0 \text{ when } x = 1.$$

$$f''(1) = -1 \cdot e^{-1} < 0.$$

So  $f$  is cc down at  $x = 1$ .

So  $f$  has a local maximum at  $x = 1$ . The maximum value is  $f(1) = 2 + e^{-1}$ .