Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #9, November 8th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (12 pts.) Evaluate the limits below. Use L'Hospital's Rule where appropriate. **Indicate when you are using L'Hospital's Rule and state explicitly the indeterminate form.**

(a.) $\lim_{\theta\to\pi/2} \frac{\cos\theta}{\sin(4\theta)} \overset{H}{=} \lim_{\theta\to\frac{\pi}{2}} \frac{-\sin\theta}{4\cos(4\theta)} = \frac{-1}{4}$

$\cos(\pi/2)=0$

$\sin(4\cdot\frac{\pi}{2})=\sin(2\pi)=0$ $\cos(2\pi)=1$

form $\frac{0}{0}$

(b.) $\lim_{x\to\infty} x^5e^{-x^4} = \lim_{x\to\infty} \frac{x^5}{e^{x^4}} \overset{H}{=} \lim_{x\to\infty} \frac{5x^4}{4x^3e^{x^4}} = \lim_{x\to\infty} \frac{5x}{4e^{x^4}}$

form $\infty\cdot 0$ form $\frac{\infty}{\infty}$ form $\frac{\infty}{\infty}$

$\overset{H}{=} \lim_{x\to\infty} \frac{5}{16x^3e^{x^4}} = 0.$

(c.) $\lim_{x\to 0}(1-3x)^{1/x} = \boxed{e^{-3}}$

Let $y=(1-3x)^{1/x}$.

So, $\ln y = \frac{1}{x}\ln(1-3x)$.

Now, $\lim_{x\to 0} \frac{\ln(1-3x)}{x} \overset{H}{=} \lim_{x\to 0} \frac{\frac{-3}{1-3x}}{1} = -3$

form $\frac{0}{0}$

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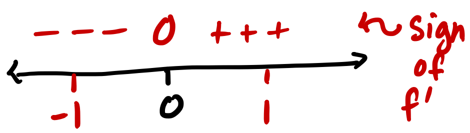
Exercise 2. (10 pts.) Use the information below to answer questions about the function $f(x)$. Make sure you answer the question! If something doesn't exist, you must explicitly state this.

$$f(x) = \frac{-1}{x^2+6}, \quad f'(x) = \frac{2x}{(x^2+6)^2}, \quad f''(x) = \frac{-6(x^2-2)}{(x^2+6)^3}.$$

(a.) Find the interval(s) where the function is increasing and the interval(s) where the function is decreasing.

$f' = 0$ when $x = 0$.

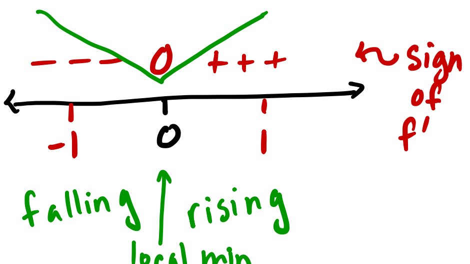
f' never undefined.



answer

f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(b.) Find the local maximum and minimum values of f and where they occur.



answer

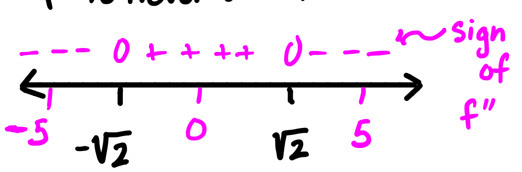
$f(0) = -1/6$ is a local minimum.

f has no local maximums.

(c.) Find the intervals of concavity and any inflection points.

$f'' = 0$ when $x = \pm\sqrt{2}$.

f'' is never undefined.



$f''(-5) = \frac{- \cdot +}{+} = -$ $\quad f''(5) = \frac{- \cdot +}{+} = -$

$f''(0) = \frac{- \cdot -}{-} = +$

$f(\sqrt{2}) = -\frac{1}{8}$

$f(-\sqrt{2}) = -1/8$

answer:

f is ccup on $(-\sqrt{2}, \sqrt{2})$ and ccdown on $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$;

inflection points at $(\sqrt{2}, -1/8)$ and $(-\sqrt{2}, -1/8)$.

Exercise 3. (2 pts.) Given $f(x) = 5 + xe^{-x}$, $f'(x) = -(x-1)e^{-x}$, and $f''(x) = (x-2)e^{-x}$, use the Second Derivative Test to identify the local minimum and maximum values of f or explain why the test is inconclusive.

$f' = 0$ when $x = 1$.

$f''(1) = -1\, e^{-1} < 0$

So f is cc down:

answer:

$f(1) = 5 + e^{-1}$ is a local maximum.

f has no local minimums.