Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #9, November 8th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (12 pts.) Evaluate the limits below. Use L'Hospital's Rule where appropriate. **Indicate when you are using L'Hospital's Rule and state explicitly the indeterminate form.**

(a.) $\lim_{x\to\infty} x^3 e^{-x^2}$ $= \lim_{x\to\infty} \frac{x^3}{e^{x^2}} \overset{H}{=} \lim_{x\to\infty} \frac{3x^2}{2x e^{x^2}} = \lim_{x\to\infty} \frac{3x}{2e^{x^2}}$

form $\infty \cdot 0$; form $\frac{\infty}{\infty}$; form $\frac{\infty}{\infty}$

$\overset{H}{=} \lim_{x\to\infty} \frac{3}{4x e^{x^2}} = 0.$

(b.) $\lim_{\theta\to\pi/2} \frac{\cos\theta}{\sin(8\theta)}$ $\overset{H}{=} \lim_{\theta\to\pi/2} \frac{-\sin\theta}{8\cos(8\theta)} = \frac{-1}{8}$

$\cos(\pi/2)=0$

$\sin(8\frac{\pi}{2})=\sin(4\pi)=0$

form $\frac{0}{0}$

(c.) $\lim_{x\to 0}(1-4x)^{1/x}$ $= \boxed{e^{-4}}$

Let $y=(1-4x)^{1/x}$.

Then, $\ln y = \frac{1}{x}\ln(1-4x)$.

Now, $\lim_{x\to 0} \frac{\ln(1-4x)}{x} \overset{H}{=} \lim_{x\to 0} \frac{\frac{-4}{1-4x}}{1} = -4$

form $\frac{0}{0}$

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Exercise 2. (2 pts.) Given $f(x) = 6 + xe^{-x}$, $f'(x) = -(x-1)e^{-x}$, and $f''(x) = (x-2)e^{-x}$, use the Second Derivative Test to identify the local minimum and maximum values of f or explain why the test is inconclusive.

$f' = 0$ when $x = 1$

$f''(1) = -1e^{-1} < 0.$

So f is ccdown at $x=1$

answer:

$f(1) = 6 + e^{-1}$ is a relative maximum.

f has no relative minimum.

Exercise 3. (10 pts.) Use the information below to answer questions about the function $f(x)$. Make sure you answer the question! If something doesn't exist, you must explicitly state this.

$$f(x) = \frac{1}{x^2+9}, \qquad f'(x) = \frac{-2x}{(x^2+9)^2}, \qquad f''(x) = \frac{6(x^2-3)}{(x^2+9)^3}.$$

(a.) Find the interval(s) where the function is increasing and the interval(s) where the function is decreasing.

$f' = 0$ when $x = 0$; f' never undefined.

$+ + +\ 0\ - - -$ ← sign of f'

$-1 \quad 0 \quad 1$

$f'(-1) = \frac{- \cdot -}{+} = +$; $f'(1) = \frac{- \cdot +}{+} = -$

answer:

f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.

(b.) Find the local maximum and minimum values of f and where they occur.

$+ + +\ 0\ - - -$ ← sign of f'

$-1 \quad 0 \quad 1$

rising ↑ falling

local max

answer

$f(0) = 1/9$ is a local maximum.

f has no local minimums.

(c.) Find the intervals of concavity and any inflection points.

$f'' = 0$ when $x^2 - 3 = 0$ or $x = \pm\sqrt{3}$

$+ + +\ 0\ - - -\ 0\ + + +$ ← sign of f''

$-5 \quad -\sqrt{3} \quad 0 \quad \sqrt{3} \quad 5$

$f''(-5) = \frac{(+)(+)}{+} = +$

$f''(0) = \frac{(+)(-)}{+} = -$

$f''(5) = \frac{(+)(+)}{+} = +$

$f(-\sqrt{3}) = 1/12$

$f(\sqrt{3}) = 1/12$

answer

$f(x)$ is concave up on $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$ and concave down on $(-\sqrt{3}, \sqrt{3})$.

Inflection points $(-\sqrt{3}, 1/12)$ and $(\sqrt{3}, 1/12)$