Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [5 points] Evaluate the limit. Show work and use proper limit notation for full credit.

$$\lim_{x\to-2}\frac{x+2}{x^2+7x+10} = \lim_{x\to-2}\frac{x+2}{(x+2)(x+5)}$$

$$= \lim_{x\to-2}\frac{1}{x+5} = \boxed{\frac{1}{3}}$$

2. [5 points] Evaluate the limit. Show work and use proper limit notation for full credit.

$$\lim_{x\to0}\frac{2-\sqrt{4+h}}{h} = \lim_{x\to0}\frac{(2-\sqrt{4+h})(2+\sqrt{4+h})}{h}$$

$$= \lim_{x\to0}\frac{4-(4+h)}{h}$$

$$= \lim_{x\to0}\frac{-h}{h} = \lim_{h\to0} -1 = \boxed{-1}$$

3. [4 points]

a. Why is the following not a true statement?:

$$\frac{5x^2-3x}{x} = 5x-3$$

The expression on the right-hand side is defined at $x=0$, but the expression on the left-hand side is not.

b. Explain why the following equation *is* correct:

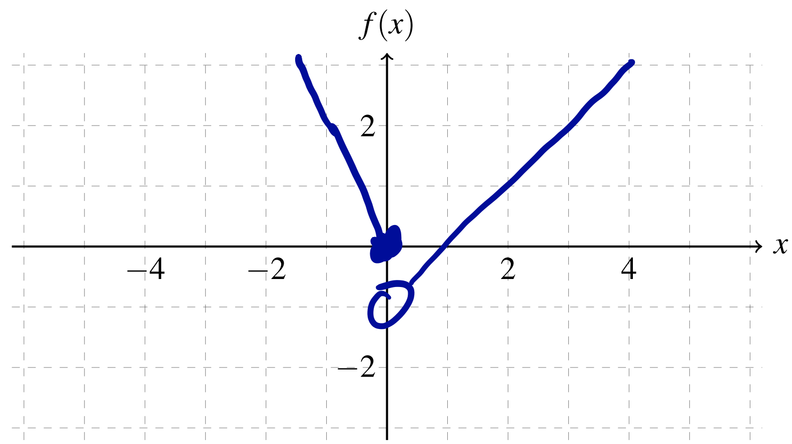
$$\lim_{x\to0}\frac{5x^2-3x}{x} = \lim_{x\to0} 5x-3$$

The expressions inside the limits are the same except at one point ($x=0$) and "limits don't care about one point."

4. [6 points] Consider the function

$$f(x) = \begin{cases} -2x & x \le 0 \\ -1+x & x > 0 \end{cases}$$

a. On the axes below, sketch a graph of $f(x)$.



b. Evaluate the limit, or explain why it does not exist:

$\lim_{x\to 0} f(x)$

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 0^-} f(x) = 0$

Since the left- and right-hand limits are different, the limit $\lim_{x\to 0} f(x)$ does not exist.

c. Is f continuous at $x = 0$? Explain using the definition of continuity.

The function is not continuous at $x=0$. We would need $\lim_{x\to 0} f(x) = f(0) = 0$. But the limit does not exist.

5. [5 points] Use the Intermediate Value Theorem to justify the claim that there exists a number x on the interval $(0,2)$ satisfying $e^x - 6x = 0$.

Let $f(x) = e^x - 6x$. Notice $f(0) = 1 > 0$, but $f(1) \approx 2.7 - 6 < 0$. Since $f(x)$ is continuous on $[0,1]$ the IVT implies for some x in $(0,1)$, $f(x) = 0$. So there is certainly x in $(0,2)$ with $f(x) = 0$.