Name: ______________________ ________ / 25

Instructor: **Bueler | Jurkowski | Maxwell**

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [5 points] A bacteria culture initially contains 100 cells and grows at a rate proportional to its population. Suppose after an hour, the population is now 300. Given that the equation $y = Ce^{kt}$ models the population at time t:

a. Determine C.

$y(t=0) = 100 = Ce^{0}$ so

$C = 100$

b. Find a simplified expression for k.

$300 = 100\, e^{k \cdot 1}$

$k = \ln\left(\frac{300}{100}\right) = \ln(3)$

2. [6 points] Suppose we are enlarging a rectangular photograph where the height is always twice the width. If the width is increasing at a rate of 2 cm/min, what is the rate at which the area of the rectangle is changing when the width is 5 cm long?

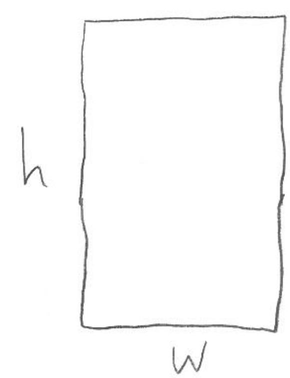
$h = 2w$

$\frac{dw}{dt} = 2 \frac{\text{cm}}{\text{min}}$

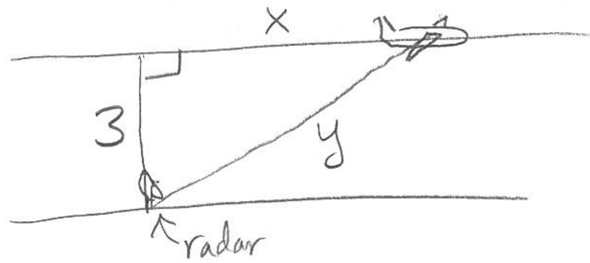
$A = wh = 2w^2$

$\frac{dA}{dt} = 4w \frac{dw}{dt}$

$\frac{dA}{dt} = 4 \cdot 5 \cdot 2 = 40 \frac{\text{cm}^2}{\text{min}}$



3. [7 points] A plane flying horizontally at an altitude of 3 km and a speed of 400 km/hr is flying directly away from a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 km away from the station. (Distance here is total distance, not horizontal distance.)



$\frac{dx}{dt} = 400 \frac{km}{hr}$

$y = $ (distance from plane to radar)

$x^2 + 3^2 = y^2 \longrightarrow 2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$

want: $\frac{dy}{dt}$ when $y=5$

(when $y=5$: $x^2+3^2=5^2$ so $x=4$)

$\Leftrightarrow \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{4}{5}\cdot 400$

$= 320 \frac{km}{hr}$

4. [7 points]

a. Find the linearization of $f(x) = \sqrt{x}$ at $a = 16$.

$f'(x) = \frac{1}{2}x^{-1/2}$

$L(x) = f(a) + f'(a)(x-a)$

$= \sqrt{16} + \frac{1}{2}\frac{1}{\sqrt{16}}(x-16)$

$= 4 + \frac{1}{8}(x-16)$

b. Use part **a.** to estimate $\sqrt{17}$. A simplified fraction or decimal will suffice.

$\sqrt{17} = f(17) \approx L(17) = 4 + \frac{1}{8}(17-16) = 4\frac{1}{8} = 4.125$