Name: _____ _____ / 25

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[5 points]** A bacteria culture initially contains 100 cells and grows at a rate proportional to its population. Suppose after an hour, the population is now 300. Given that the equation $y = Ce^{kt}$ models the population at time t:

   a. Determine C.

   $$100 = y(t=0) = Ce^{0}$$
   $$\boxed{C = 100}$$

   b. Find a simplified expression for k.

   $$300 = 100e^{k \cdot 1}$$
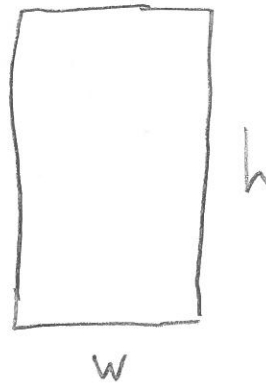 $$\boxed{k = \ln\left(\frac{300}{100}\right) = \ln 3}$$

2. **[6 points]** Suppose we are enlarging a rectangular photograph where the height is always twice the width. If the width is increasing at a rate of 3 cm/min, what is the rate at which the area of the rectangle is changing when the width is 4 cm long?

   $$h = 2w$$
   $$A = wh = 2w^2$$
   $$\frac{dw}{dt} = 3 \frac{cm}{min}$$
   $$want: \frac{dA}{dt} \text{ when } w = 4$$

   $$\boxed{\frac{dA}{dt} = 4w\frac{dw}{dt} = 4 \cdot 4 \cdot 3 = 48 \frac{cm^2}{min}}$$

3. [7 points]

a. Find the linearization of $f(x) = \sqrt{x}$ at $a = 16$.

$f'(x) = \frac{1}{2}x^{-1/2}$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= \sqrt{16} + \frac{1}{2\sqrt{16}}(x-16)$$
$$= 4 + \frac{1}{8}(x-16) = 2 + \frac{1}{8}x$$

b. Use part **a.** to estimate $\sqrt{15}$. A simplified fraction or decimal will suffice.

$$\sqrt{15} = f(15) \approx L(15) = 4 + \frac{1}{8}(15-16)$$
$$= 4 - \frac{1}{8} = 3.875 = \frac{31}{8}$$

4. [7 points] A plane flying horizontally at an altitude of 4 km and a speed of 400 km/hr is flying directly away from a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 km away from the station. (Distance here is total distance, not horizontal distance.)

ground
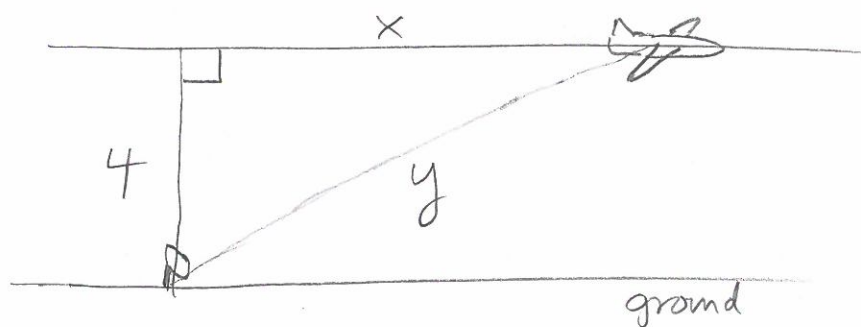
$$\frac{dx}{dt} = 400 \frac{km}{hr} \qquad \text{want } \frac{dy}{dt} \text{ when } y = 5$$

$x = 3$ when $y = 5$

$$x^2 + 4^2 = y^2$$
$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{3}{5} \cdot 400$$
$$= 3 \cdot 80 = 240 \frac{km}{hr}$$