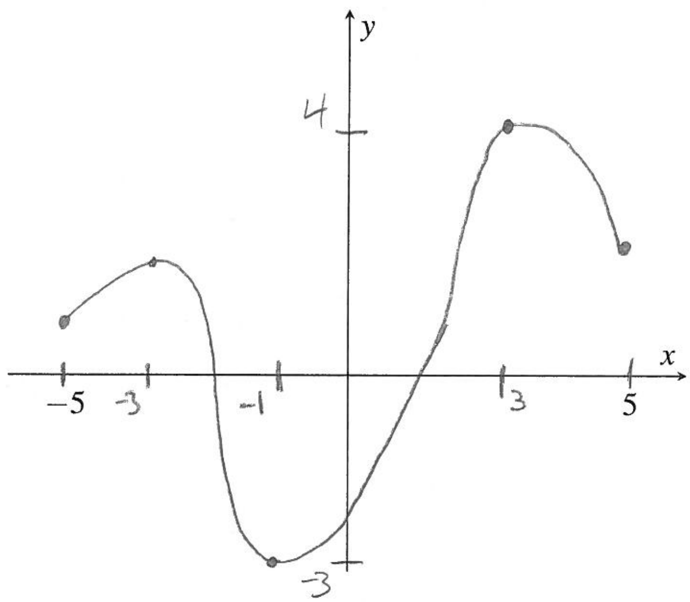
Name: ______________ ______ / 25

Instructor: Bueler | Jurkowski | Maxwell

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[5 points]** Sketch a function on $[-5,5]$ that has an absolute maximum value of 4 at $x=3$, an absolute minimum value of -3 at $x=-1$, and a local maximum at $x=-3$. You should appropriately label notable values on the x- and y-axes for full credit.



one possibility

2. **[4 points]** Find all critical numbers (a.k.a. critical points) of the function $f(x) = \sqrt[3]{9-x^2}$. Be careful!

[the domain of f(x) is $(-\infty,\infty)$ since $\sqrt[3]{z}$ is defined for any z]

$$f'(x) = \frac{1}{3}(9-x^2)^{-2/3}(-2x) = \frac{-2x}{(9-x^2)^{2/3}}$$

$x=0$ or $9-x^2=0$

$x=\pm 3$

critical numbers are $x=-3, 0, 3$

3. [8 points] Find the maximum and minimum values of the function $f(x) = x + \frac{4}{x}$ on the interval $[1,5]$.

$f(x)$ is continuous on $[1,5]$

$f'(x) = 1 - \frac{4}{x^2} = 0 \iff x = \pm 2$

only $x = +2$ is in $[1,5]$

x	$f(x)$
1	5
2	4
5	$5 + \frac{4}{5} = 5.8$

absolute maximum at $f(5) = 5.8$

absolute minimum at $f(2) = 4$

4. [8 points] Suppose f is continuous on $[-2,2]$ and has a derivative at each point in $(-2,2)$. Suppose $f(-2) = 4$ and $f(2) = -6$.

a. What specifically does the Mean Value Theorem let you conclude?

there is c in $(-2,2)$ so that

$$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{-6-4}{4} = -\frac{5}{2}$$

b. Draw a diagram that illustrates the Mean Value Theorem for this problem. Your illustration should include a tangent line somewhere.