Name: _____ _____ / 25

Instructor: **Bueler | Jurkowski | Maxwell**

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [8 points] The function $f(x)$ with domain $(-\infty,\infty)$ has the following properties.

1. $f(0)=5; \quad f(2)=0$
2. $f'(x)=0$ at $x=0$ and $x=2$, and $f'(x)$ otherwise has signs:

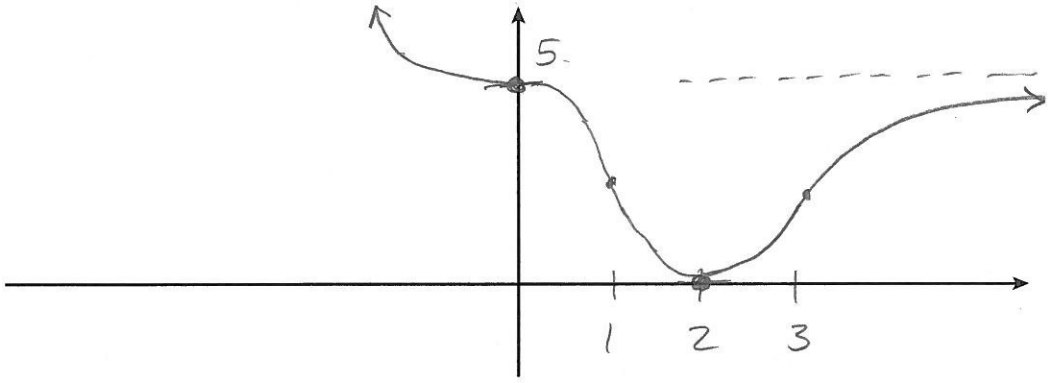
$f'(x)$: $-$ on $(-\infty,0)$, $-$ on $(0,2)$, $+$ on $(2,\infty)$; zeros at 0, 2

3. $f''(x)=0$ at $x=0$, 1 and 3 and $f''(x)$ otherwise has signs:

$f''(x)$: $+$ on $(-\infty,0)$, $-$ on $(0,1)$, $+$ on $(1,3)$, $-$ on $(3,\infty)$; zeros at 0, 1, 3

4. $\lim_{x\to-\infty} f(x)=\infty; \quad \lim_{x\to\infty} f(x)=5$

Make a sketch of the graph of the function on the axes below.



2. [4 points] Compute the following limits.

a. $\lim_{x\to0}\dfrac{e^{\pi x}-1}{\sin x}$.

$\overset{\text{L'H}}{=}$ $\dfrac{0}{0}$ $\lim_{x\to0}\dfrac{\pi e^{\pi x}}{\cos x}=\dfrac{\pi\cdot1}{1}=\pi$

b. $\lim_{x\to\infty}\dfrac{\ln x}{x^2}$.

$\overset{\text{L'H}}{=}$ $\dfrac{\infty}{\infty}$ $\lim_{x\to\infty}\dfrac{\frac{1}{x}}{2x}=\lim_{x\to\infty}\dfrac{1}{2x^2}=0$

3. [13 points] Consider the function $f(x) = \dfrac{\ln x}{x^2}$. We have computed for you

$$f'(x) = \frac{1-2\ln x}{x^3}; \qquad f''(x) = \frac{6\ln x - 5}{x^4}.$$

a. Find the domain of $f(x)$.

$(0, \infty)$

b. Find the vertical and horizontal asymptotes. [Can **2b.** from the previous page help?]

$x=0$ is vertical because $\lim_{x\to 0^+} \frac{\ln x}{x^2} = -\infty$

$y=0$ is horizontal because $\lim_{x\to\infty} \frac{\ln x}{x^2} = 0$

c. Find the single critical point c and the intervals where $f(x)$ is increasing and decreasing.

$f'(x) = \frac{1-2\ln x}{x^3} = 0$ $\quad c = e^{1/2}$

$1-2\ln x = 0 \Leftrightarrow \ln x = \frac{1}{2}$

increasing: $(0, c)$

decreasing: (c, ∞)

d. Determine whether $f(x)$ has a local minimum, maximum, or neither at $x = c$ using the first derivative test.

local maximum at $x = c = e^{1/2}$

by first derivative test

e. Find the intervals where $f(x)$ is concave up and concave down.

$6\ln x - 5 = 0$

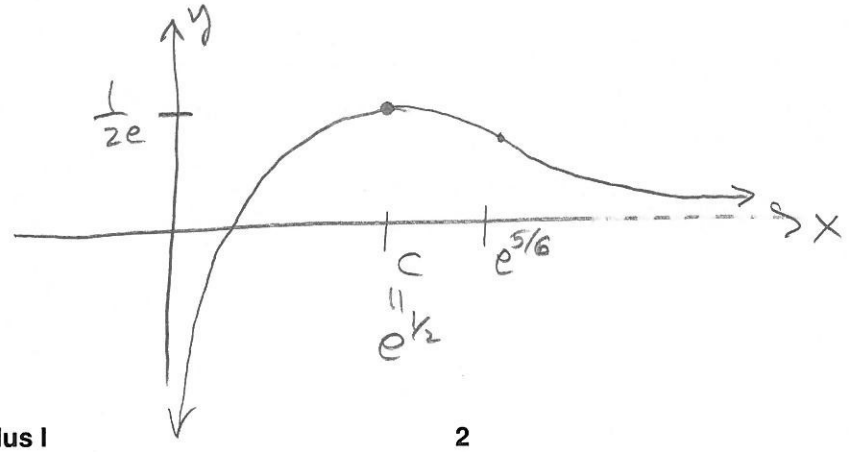
$\ln x = \frac{5}{6}$

$x = e^{5/6}$

Concave down: $(0, e^{5/6})$

Concave up: $(e^{5/6}, \infty)$

f. Using the information above, sketch the graph of $f(x)$, making sure to label important points.



$f(c) = \frac{\ln(e^{1/2})}{e} = \frac{1}{2e}$