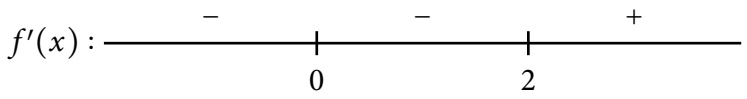
Name: _____ _____ **/ 25**

Instructor: Bueler | Jurkowski | Maxwell

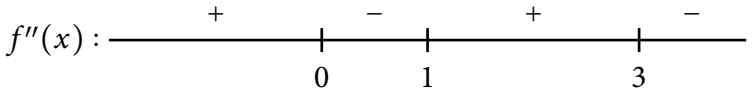
There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [8 points] The function $f(x)$ with domain $(-\infty,\infty)$ has the following properties.

1. $f(0) = 5; \quad f(2) = 0$
2. $f'(x) = 0$ at $x = 0$ and $x = 2$, and $f'(x)$ otherwise has signs:

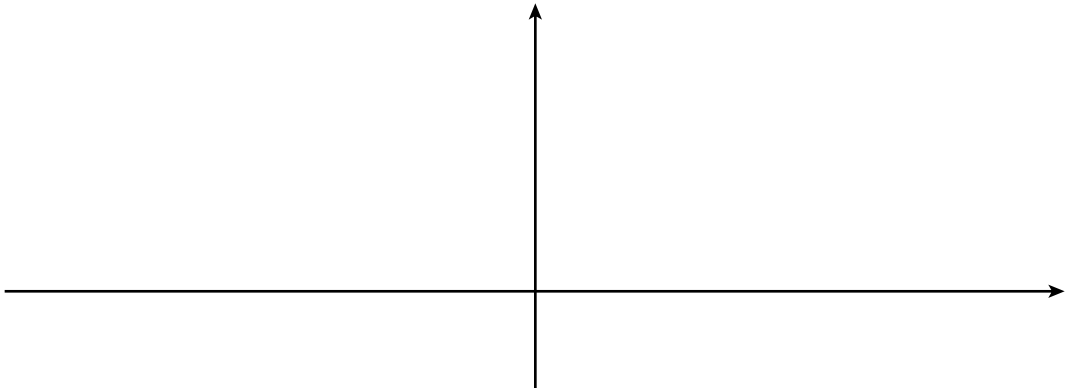


3. $f''(x) = 0$ at $x = 0$, 1 and 3 and $f''(x)$ otherwise has signs:



4. $\lim_{x\to-\infty} f(x) = \infty; \quad \lim_{x\to\infty} f(x) = 5$

Make a sketch of the graph of the function on the axes below.



2. [4 points] Compute the following limits.

a. $\lim_{x\to 0} \dfrac{e^{\pi x} - 1}{\sin x}$.

b. $\lim_{x\to\infty} \dfrac{\ln x}{x^2}$.

3. [13 points] Consider the function $f(x) = \dfrac{\ln x}{x^2}$. We have computed for you

$$f'(x) = \frac{1-2\ln x}{x^3}; \qquad f''(x) = \frac{6\ln x - 5}{x^4}.$$

a. Find the domain of $f(x)$.

b. Find the vertical and horizontal asymptotes. [Can **2b.** from the previous page help?]

c. Find the single critical point c and the intervals where $f(x)$ is increasing and decreasing.

d. Determine whether $f(x)$ has a local minimum, maximum, or neither at $x = c$ using the first derivative test.

e. Find the intervals where $f(x)$ is concave up and concave down.

f. Using the information above, sketch the graph of $f(x)$, making sure to label important points.