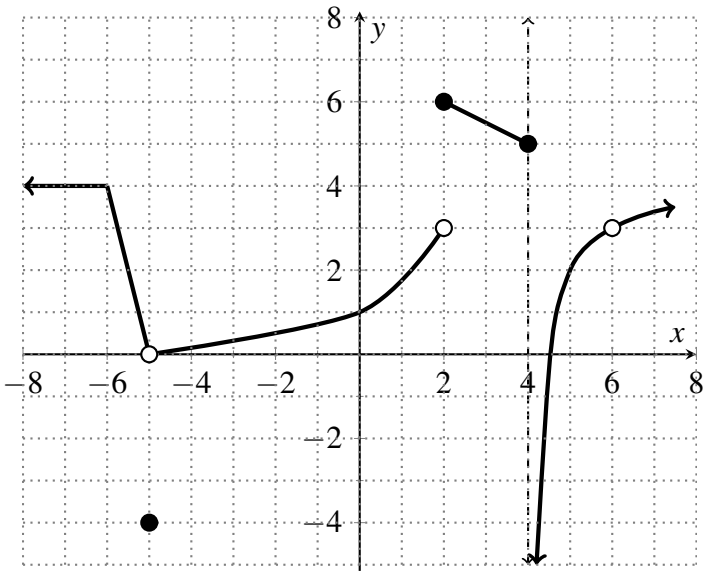
Name: _____ _____ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [2 points] Use the graph of the function of $f(x)$ to find **all** x-values where $f(x)$ fails to be continuous.



Answer: $x =$ $-5, 2, 4, 6$

2. [4 points]

a. What is wrong with the following equation? $\dfrac{x^4 - 4x}{x} = x^3 - 4$

It is false when $x=0$ because the left is undefined and the right is 1.

b. In view of part a, explain why the following equation is correct. $\lim\limits_{x\to 0} \dfrac{x^4 - 4x}{x} = \lim\limits_{x\to 0} x^3 - 4$

Because the limit does not care what happens right at $x=0$. The functions are the same for all other values.

3. [4 points] Explain why the function $f(x) = \begin{cases} 3\cos x & x < 0 \\ -2 & x = 0 \\ 4x - 2 & x > 0. \end{cases}$ fails to be continuous at $x = 0$.

$\lim\limits_{x\to 0^-} 3\cos x = 3$ but $\lim\limits_{x\to 0^+} 4x-2 = -2$.

So $\lim\limits_{x\to 0} f(x)$ does not exist.

4. [12 points] Evaluate each limit below, if it exists. Show your work to receive full credit. If the limit is infinite, say so; don't just write "DNE".

a. $\lim_{x\to -3} \dfrac{x^2+x-6}{15+2x-x^2}$.

$$= \lim_{x\to -3} \frac{(x+3)(x-2)}{-(x^2-2x-15)} = \lim_{x\to -3} \frac{(x+3)(x-2)}{-(x+3)(x-5)}$$

$$= \lim_{x\to -3} \frac{-(x-2)}{x-5} = \frac{-(-5)}{-8} = -\frac{5}{8}$$

b. $\lim_{h\to 5^-} \dfrac{h-5}{4|h|-20}$

$$= \lim_{h\to 5^-} \frac{h-5}{4(|h|-5)} = \lim_{h\to 5^-} \frac{h-5}{4(h-5)} = \lim_{h\to 5^-} \frac{1}{4} = \frac{1}{4}$$

c. $\lim_{x\to 3^-} \left(\dfrac{1}{x-3} - \dfrac{1}{x(x-3)}\right)$

$$= \lim_{x\to 3^-} \frac{x-1}{x(x-3)} = -\infty$$

as $x\to 3^-$, $x-1>0$, $x>0$ and $x-3<0$.

Also, as $x\to 3^-$, $x-3\to 0^-$

5. [3 points] What property of the natural log function allows you to move the limit inside the function, as done below?

$$\lim_{x\to 5} \left(\ln\left(x^2+16\right)\right) = \ln\left(\lim_{x\to 5}(x^2+16)\right)$$

$y = \ln x$ is continuous where it is defined.