Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[15 points]** Find the derivatives of each of the following. You do not need to simplify your answer.

a. $h(\theta) = e^2 \sec(\theta) + \cot(\theta)$

$h'(\theta) = e^2 \sec\theta \tan\theta - \csc^2\theta$

b. $y = \cos(5x^2)$

$\frac{dy}{dx} = \left(-\sin(5x^2)\right)(10x)$

c. $f(x) = \dfrac{\tan(x)}{x - 3\sin(x)}$

$f'(x) = \dfrac{(x-3\sin x)(\sec^2 x) - (\tan x)(1-3\cos x)}{(x-3\sin x)^2}$

d. $f(q) = q^3 e^{5q+6}$

$f'(q) = 3q^2 \left(e^{5q+6}\right)(5) + q^3 \left(e^{5q+6}\right)(5)$

e. $k(t) = (\sqrt[5]{t} - 7t + 3)^5$ $= \left(t^{1/5} - 7t + 3\right)^5$

$k'(t) = 5\left(t^{1/5} - 7t + 3\right)^4 \left(\frac{1}{5} t^{-4/5} - 7\right)$

2. [4 points] Find an x-value such that the function $f(x) = 2x + \cos(4x)$ has a horizontal tangent line. (You do not have to find *every* value. Simply find one.)

$f'(x) = 2 - 4\sin(4x) = 0$

$2 = 4\sin(4x)$

$\sin(4x) = \frac{1}{2}$

$\sin(\theta) = \frac{1}{2}$ when $\theta = \frac{\pi}{6}$

So we need

$4x = \pi/6$

$x = \pi/24$

3. [6 points] In a certain experiment involving bacteria, the number N of bacteria in a culture after t days is modeled by the function

$$N(t) = 900\left(1 + \frac{3}{(t^2+1)^2}\right). = 900\left(1 + 3(t^2+1)^{-2}\right)$$

a. How many bacteria are in the culture at the beginning of the experiment?

beginning means $t=0$.

$N(0) = 900(1+3) = 900 \cdot 4 = 3600$ bacteria

b. Compute $N'(t)$. (You do not need to simplify, but you may if you choose.)

$$N'(t) = 900\left(0 + 3(-2)(t^2+1)^{-3}(2t)\right) = \frac{-12(900)t}{(t^2+1)^3}$$

c. After one day, is the number of bacteria in the culture **increasing** or **decreasing**, and how do you know? (Justify your answer; an answer with no justification will receive no credit.)

The number of bacteria is decreasing

because $N'(1) < 0$.