Name: _____ _____ / **25**

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[8 points]** A small bug is crawling along a branch of a tree. The bug's distance, in millimeters, from the trunk after t seconds is given by the function

$$s(t) = 6t - \frac{9}{2}t^2 + t^3.$$

a. What is the velocity of the bug at time t?

$v(t) = s'(t) = 6 - 9t + 3t^2 = 3(t^2 - 3t + 2) = 3(t-2)(t-1)$

b. When is the bug at rest?

Want t when $v=0$. So $t = 2$ sec and $t = 1$ sec

c. What is the acceleration of the bug when it's at rest?

$a(t) = s''(t) = -9 + 6t$

$a(1) = -9+6 = -3$ mm/s^2 $a(2) = -9+12 = 3$ mm/sec^2

d. At time $t = 3$, is the bug moving toward the trunk or away from the trunk? Justify your answer.

$v(3) = 3(1)(2) = 6 > 0$

The bug is moving away because $v(3) > 0$.

2. **[2 points]** Let P denote the population of an invasive species of fish that is growing over time, t. Suppose the population P grows at a rate proportional its size. What can you say about the function $P(t)$?

- $P(t) = P_0 e^{kt}$.
- or $P(t)$ grows exponentially

3. [6 points] The edge of a cube was found to be 5 meters with a possible error in measurement of 0.1 meter. Use differentials to estimate the maximum possible error in computing the surface area of the cube. Include units with your answer.

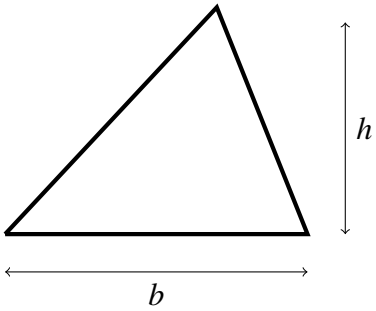
$s = 5 \quad ds = \frac{1}{10}$

$A = 6s^2$

$dA = 6 \cdot 2s \cdot ds$

$dA = 12 \cdot 5 \cdot \frac{1}{10} = \boxed{6 \text{ m}^2}$

4. [9 points] The altitude (height, h) of a triangle is increasing at at rate of 3 cm/sec while the area of the triangle is decreasing at a rate of 1 cm^2/sec. At what rate is the base, b, of the triangle changing with the altitude is 20 cm and the area is 100 cm^2? Include units with your answer.



1. Given $\frac{dh}{dt} = 3$ and $\frac{dA}{dt} = -1$

2. Want $\frac{db}{dt}$.

3. Use $A = \frac{1}{2}bh$

aside: When $h = 20$ and $A = 100$,
$100 = \frac{1}{2} b \cdot 20$.
So $b = 10$.

4. $\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt} h + b \cdot \frac{dh}{dt}\right)$

5. $-1 = \frac{1}{2}\left(\frac{db}{dt} \cdot 20 + 10 \cdot 3\right)$

$-2 = 20\frac{db}{dt} + 30$

$\frac{db}{dt} = \frac{-32}{20} = \frac{-8}{5}$ cm/sec