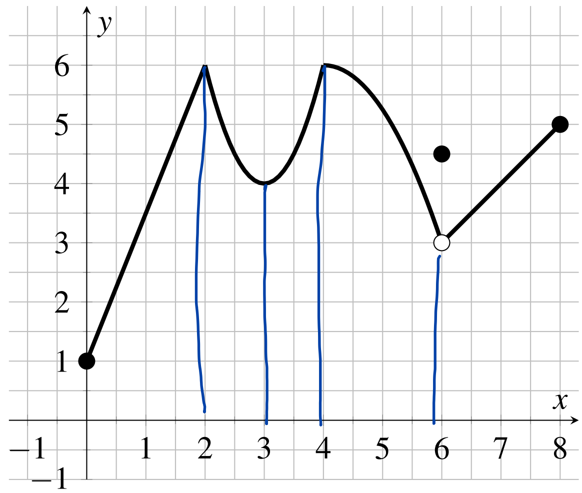
Name: Solutions ______ / 25

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [4 points] Use the graph to determine all the absolute and local maximum and minimum values of the function. If a value does not exist, write DNE.

	y-value	occurs at $x =$
local max (list all)	6 5.5	2, 4 6
local min (list all)	4	3
absolute max	6	2, 4
absolute min	1	0



Note I am not considering endpoints to be local optima. But we're not taking off points if you list them.

2. [7 points] Find the absolute maximum and absolute minimum values of

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the interval $[0,3]$, and the x-values where they occur.

$f'(x) = 6x^2 - 6x - 12$. Note $f'(x)$ is never undefined, because it is a polynomial.

$f'(x) = 0 \Rightarrow 6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2$ or $x = -1$. Only $\underline{\underline{x=2}}$ is in the domain.

Test values:

x	$f(x)$
0	1
2	-19
3	-8

$f(0) = 1$

$f(2) = 2(8) - 3(4) - 12(2) + 1$
$= 16 - 12 - 24 + 1 = -20 + 1$
$= -19$

$\begin{array}{r} 36 \\ -\ 16 \\ \hline 20 \end{array}$

$f(3) = 2(27) - 3(9) - 12(3) + 1$
$= 2(27) - 27 - 36 + 1$
$= 27 - 36 + 1 = -8$

$\begin{array}{r} 36 \\ -\ 27 \\ \hline 9 \end{array}$

Absolute Maximum: $y =$ 1 at $x =$ 0

Absolute Minimum: $y =$ −19 at $x =$ 2

3. [8 points]

Consider the function $f(x)$ shown on the graph below, on the interval $[0,2]$. It has the property that $f(2)=0$ and $f(0)=\frac{3}{2}$.

a. Fill in the blanks: The function $f(x)$ satisfies the hypotheses of the Mean Value Theorem, which means that $f(x)$ is continuous on $[0,2]$ and differentiable on $(0,2)$.

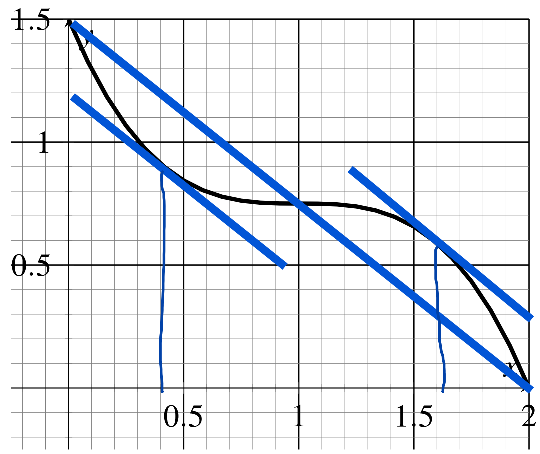
b. What can we conclude about the function $f(x)$, by the Mean Value Theorem? (That is, state the conclusion of the Mean Value Theorem, specified to this function.)

There exists $c \in (0,2)$ s.t.

$$f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{0-3/2}{2}$$

$$= -\frac{3}{4}$$

c. The graph of $f(x)$ is shown below. $\boxed{\text{Add lines to the graph}}$ to illustrate what the Mean Value Theorem says about this function. Then use the the graph to estimate the value(s) of c whose existence is predicted by the Mean Value Theorem.



Estimated value(s) of c (to the nearest tenth) predicted by MVT (list all): $c = 0.4$ and $c = 1.6$

4. [6 points] Find the critical numbers (critical points) of the function

$$g(x) = \sqrt[3]{x^2-4}. = (x^2-4)^{1/3}$$

$$g'(x) = \frac{1}{3}(x^2-4)^{-2/3}(2x) = \frac{2x}{3\sqrt[3]{(x^2-4)^2}}$$

$g'(x) = 0$ when $x = 0$.

$g'(x)$ DNE when $x^2-4=0 \Rightarrow x = 2$ or $x = -2$

Critical points: $x =$ $-2, 0, 2$