Math 251: Quiz 8 November 5, 2019

Name: Solutions ______ / 25

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [9 points] The function $j(x)$ and its first two derivatives are given below. Use them to answer parts (a)-(d).

$$j(x) = \frac{(x+1)^2}{x^2+1}, \qquad j'(x) = \frac{-2(x-1)(x+1)}{(x^2+1)^2}, \qquad j''(x) = \frac{4x(x^3+3)}{(x^2+1)^3}$$

a. Does $j(x)$ have any vertical asymptotes? Justify your answer.

No. $x^2+1=0$ has no solution.

b. Does $j(x)$ have any horizontal asymptotes? Justify your answer.

Yes. $y=1$. Reason: $\lim_{x\to\infty} \frac{(x+1)^2}{x^2+1} = \lim_{x\to\infty} \frac{1+\frac{2}{x}+\frac{1}{x^2}}{1+\frac{1}{x^2}} = 1$

c. Determine the intervals on which $j(x)$ is increasing or decreasing. Show your work to receive credit.

crit. pts: $x=\pm1$

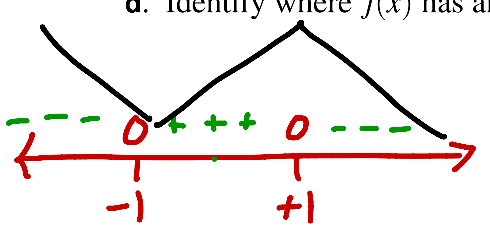
$- - -\ 0\ + + +\ 0\ - - -$ ← sign of $j'(x)$

$-10 \quad -1 \quad 0 \quad +1 \quad 10$ ← sample points

$(-)(-)(-) \quad (-)(-)(+) \quad (-)(+)(+)/(+)$

answer: $j(x)$ is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)\cup(1,\infty)$

d. Identify where $j(x)$ has any local minimums or local maximums.



answer:
local min at $x=-1$
local max at $x=1$

2. [8 points] Find the limit.

a. $\lim_{t\to0} \frac{e^{17t}-1}{\sin(2t)} \overset{H}{=} \lim_{t\to0} \frac{17e^{17t}}{2\cos(2t)} = \frac{17}{2}$

form $\frac{0}{0}$

b. $$\underbrace{\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)}_{\text{form } \infty-\infty} \overset{\text{common denominator}}{=} \underbrace{\lim_{x\to 0^+}\frac{e^x-1-x}{x(e^x-1)}}_{\text{form } \frac{0}{0}} \overset{H}{=} \underbrace{\lim_{x\to 0^+}\frac{e^x-1}{e^x-1+xe^x}}_{\text{form } \frac{0}{0}}$$

$$\overset{H}{=} \lim_{x\to 0^+}\frac{e^x}{e^x+e^x+xe^x} = \frac{1}{2}$$

3. [8 points] On the axes below, sketch the graph of a function that satisfies **all** of the given conditions. Label on your sketch any local maximums, any local minimums, and any inflection points.

a. $k(x)$ is continuous and differentiable for all real numbers. ← smooth, all 1 piece

$\boxed{\textbf{b.}}$ $k(0) = 2$ point $(0,2)$

c. The table below gives information about the sign of first derivative of $k(x)$.

horizontal tangent

x	$-\infty < x < -4$	$x = -4$	$-4 < x < 0$	$x = 0$	$0 < x < \infty$
$k'(x)$	$-$	0	$+$	0	$+$

decreasing to $x=-4$ increasing after

d. The table below gives information about the sign of second derivative of $k(x)$.

x	$-\infty < x < -1$	$x = -1$	$-1 < x < 0$	$x = 0$	$0 < x < \infty$
$k''(x)$	$+$	0	$-$	0	$+$

ccup everywhere except between $x=-1$ and $x=0$