

Name: _____

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25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [9 points] The function $j(x)$ and its first two derivatives are given below. Use them to answer parts (a)-(d).

$$j(x) = \frac{(x+1)^2}{x^2+1}, \quad j'(x) = \frac{-2(x-1)(x+1)}{(x^2+1)^2}, \quad j''(x) = \frac{4x(x^3+3)}{(x^2+1)^3}$$

- a. Does $j(x)$ have any vertical asymptotes? Justify your answer.
- b. Does $j(x)$ have any horizontal asymptotes? Justify your answer.
- c. Determine the intervals on which $j(x)$ is increasing or decreasing. Show your work to receive credit.
- d. Identify where $j(x)$ has any local minimums or local maximums.

2. [8 points] Find the limit.

a. $\lim_{t \rightarrow 0} \frac{e^{17t} - 1}{\sin(2t)}$

b. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

3. [8 points] On the axes below, sketch the graph of a function that satisfies **all** of the given conditions. Label on your sketch any local maximums, any local minimums, and any inflection points.

- a. $k(x)$ is continuous and differentiable for all real numbers.
- b. $k(0) = 2$
- c. The table below gives information about the sign of first derivative of $k(x)$.

| | | | | | |
|---------|--------------------|----------|--------------|---------|------------------|
| x | $-\infty < x < -4$ | $x = -4$ | $-4 < x < 0$ | $x = 0$ | $0 < x < \infty$ |
| $k'(x)$ | - | 0 | + | 0 | + |

- d. The table below gives information about the sign of second derivative of $k(x)$.

| | | | | | |
|----------|--------------------|----------|--------------|---------|------------------|
| x | $-\infty < x < -1$ | $x = -1$ | $-1 < x < 0$ | $x = 0$ | $0 < x < \infty$ |
| $k''(x)$ | + | 0 | - | 0 | + |

