Math 251: Quiz 8 November 5, 2019

Name: Solutions ____ / 25

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. **[9 points]** The function $j(x)$ and its first two derivatives are given below. Use them to answer parts (a)-(d).

$$j(x) = \frac{(x+1)^2}{x^2+2}, \qquad j'(x) = \frac{-2(x-2)(x+1)}{(x^2+2)^2}, \qquad j''(x) = \frac{2(2x^3-3x^2-12x+2)}{(x^2+2)^3}$$

a. Does $j(x)$ have any vertical asymptotes? Justify your answer.

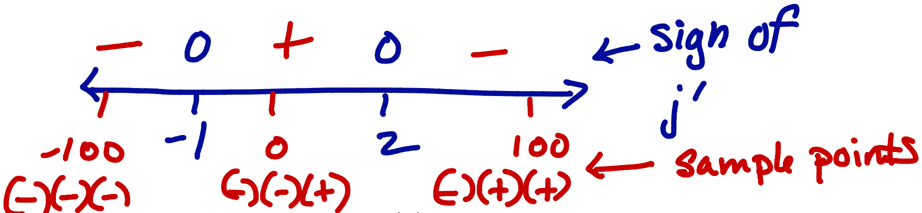
No. $x^2+2=0$ has no solution

b. Does $j(x)$ have any horizontal asymptotes? Justify your answer.

Yes. $y=1$. Reason: $\lim\limits_{x\to\infty} \frac{(x+1)^2}{x^2+2} = \lim\limits_{x\to\infty} \frac{1+\frac{2}{x}+\frac{1}{x^2}}{1+\frac{2}{x^2}} = 1$

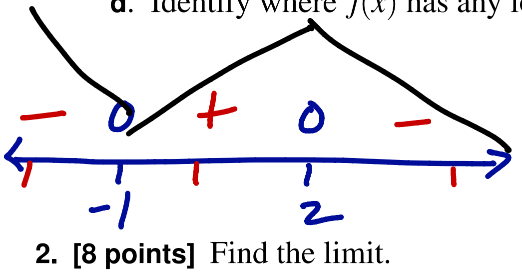
c. Determine the intervals on which $j(x)$ is increasing or decreasing. Show your work to receive credit.

Critical points: $x=2, x=-1$



answer:
$j(x)$ is increasing on $(-1,2)$
and decreasing $(-\infty,-1) \cup (2,\infty)$

d. Identify where $j(x)$ has any local minimums or local maximums.



local min at $x=-1$
local max at $x=2$

2. **[8 points]** Find the limit.

a. $\lim\limits_{t\to 0} \frac{e^{13t}-1}{\sin(4t)} \overset{H}{=} \lim\limits_{t\to 0} \frac{13e^{13t}}{4\cos(4t)} = \frac{13}{4}$

form $\frac{0}{0}$

form $\infty - \infty$ common denominator form $\frac{0}{0}$ form $\frac{0}{0}$

b. $\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right) = \lim_{x\to 0^+} \frac{e^x - 1 - x}{x(e^x-1)} \overset{H}{=} \lim_{x\to 0^+} \frac{e^x - 1}{1\cdot(e^x-1) + xe^x}$

$\overset{H}{=} \lim_{x\to 0^+} \frac{e^x}{e^x + e^x + xe^x} = \frac{1}{2}$

3. [8 points] On the axes below, sketch the graph of a function that satisfies **all** of the given conditions. Label on your sketch any local maximums, any local minimums, and any inflection points.

a. $k(x)$ is continuous and differentiable for all real numbers. ← smooth + all 1 piece

✓ **b.** $k(0) = 2$ point (0,2)

c. The table below gives information about the sign of first derivative of $k(x)$.

local max

x	$-\infty < x < -2$	$x = -2$	$-2 < x < 1$	$x = 1$	$1 < x < \infty$
$k'(x)$	$+$	0	$-$	0	$-$

d. The table below gives information about the sign of second derivative of $k(x)$.

x	$-\infty < x < 0$	$x = 0$ (Ipt)	$0 < x < 0$	$x = 1$ (Ipt)	$1 < x < \infty$
$k''(x)$	$-$	0	$+$	0	$-$