

Name: \_\_\_\_\_

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25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [9 points] The function  $j(x)$  and its first two derivatives are given below. Use them to answer parts (a)-(d).

$$j(x) = \frac{(x+1)^2}{x^2+2}, \quad j'(x) = \frac{-2(x-2)(x+1)}{(x^2+2)^2}, \quad j''(x) = \frac{2(2x^3 - 3x^2 - 12x + 2)}{(x^2+2)^3}$$

- a. Does  $j(x)$  have any vertical asymptotes? Justify your answer.
- b. Does  $j(x)$  have any horizontal asymptotes? Justify your answer.
- c. Determine the intervals on which  $j(x)$  is increasing or decreasing. Show your work to receive credit.
- d. Identify where  $j(x)$  has any local minimums or local maximums.

2. [8 points] Find the limit.

a.  $\lim_{t \rightarrow 0} \frac{e^{13t} - 1}{\sin(4t)}$

b.  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right)$

3. [8 points] On the axes below, sketch the graph of a function that satisfies **all** of the given conditions. Label on your sketch any local maximums, any local minimums, and any inflection points.

- a.  $k(x)$  is continuous and differentiable for all real numbers.
- b.  $k(0) = 2$
- c. The table below gives information about the sign of first derivative of  $k(x)$ .

$x$	$-\infty < x < -2$	$x = -2$	$-2 < x < 1$	$x = 1$	$1 < x < \infty$
$k'(x)$	+	0	-	0	-

- d. The table below gives information about the sign of second derivative of  $k(x)$ .

$x$	$-\infty < x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x < \infty$
$k''(x)$	-	0	+	0	-

