Name: _____ _____ / 25

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify unless asked, but show all work and use proper notation for full credit.

1. **[5 points]** Oil leaked out from a tanker at a rate of $r(t)$ liters per hour (L/h). Every two hours, the rate of flow out of the tanker was measured as shown in the table. Estimate the total amount of oil that leaked from the tanker over the 8 hours. ~~Justify your answer.~~

t (in hours)	0	2	4	6	8
$r(t)$ (in liters/hour)	0	5	10	15	25

total (using RH endpts) $= 2(5) + 2(10) + 2(15) + 2(25)$

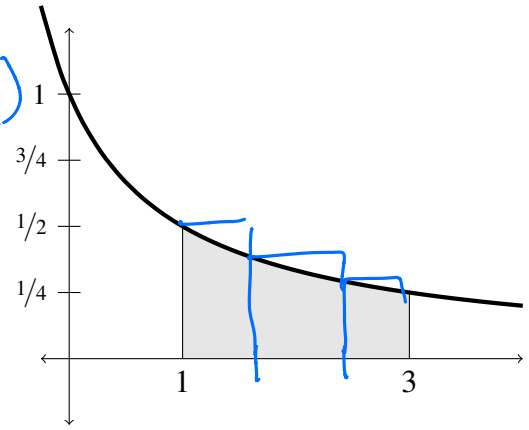
$= 10 + 20 + 30 + 50$

$= 110$ Liters.

2. **[6 points]**

Given $f(x) = \frac{1}{1+x}$, approximate the area (shaded in gray) bounded between the curve and the x-axis and the lines $x = 1$ and $x = 3$ using three LEFT-HAND rectangles. **Draw and shade in the rectangles you are using on the graph.** Show your computation clearly, and simplify your answer.

(a) Width of each rectangle = _____ $\frac{3-1}{3} = \frac{2}{3}$



area $= \frac{2}{3}\left(\frac{1}{1+1}\right) + \frac{2}{3}\left(\frac{1}{1+1+2/3}\right) + \frac{2}{3}\left(\frac{1}{1+1+4/3}\right)$

$= \frac{2}{3}\left(\frac{1}{2}\right) + \frac{2}{3}\left(\frac{1}{2+\frac{2}{3}}\right) + \frac{2}{3}\left(\frac{1}{2+\frac{4}{3}}\right)$

$= \frac{2}{3}\left(\frac{1}{2}\right) + \frac{2}{3}\left(\frac{1}{8/3}\right) + \frac{2}{3}\left(\frac{1}{10/3}\right)$

$= \frac{2}{3}\left(\frac{1}{2}\right) + \frac{2}{3}\left(\frac{3}{8}\right) + \frac{2}{3}\left(\frac{3}{10}\right)$

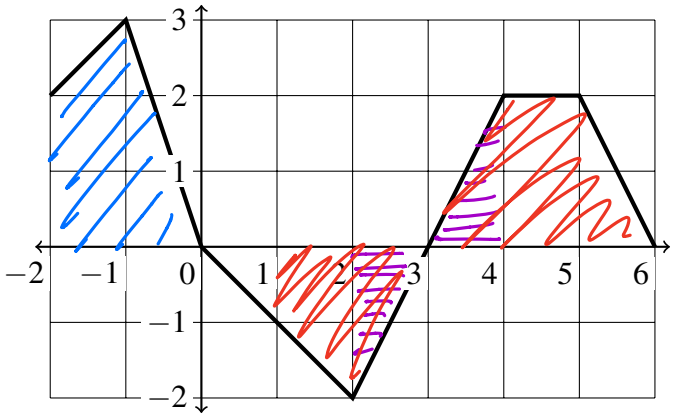
$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20}{60} + \frac{15}{60} + \frac{12}{60} = \frac{27+20}{60} = \frac{47}{60}$

(b) Approximate area = _____ $47/60$

(c) Is your answer an overestimate or an underestimate for the actual area, and why?

Overestimate since each rectangle overhangs the actual area

3. [3 points] Using the graph of $f(x)$ shown to the right and geometry, calculate exactly each of the following quantities:



a. $\int_{-2}^{0} f(x)\,dx =$ ~~5~~ 4

$= 2 + \frac{1}{2} + \frac{1}{2}(3) =$ ~~5~~ 4

b. $\int_{2}^{4} f(x)\,dx =$ 0

c. $\int_{1}^{5} f(x)\,dx =$ ~~6~~ 6 1½

$-\left(1 + \frac{1}{2} + 1\right) + 4 = +1\,\frac{1}{2}$

4. [8 points] If $\int_1^5 f(x)\,dx = 5$, $\int_1^8 f(x)\,dx = 8$ and $\int_1^5 g(x)\,dx = -3$, compute the following quantities:

a. $\int_1^5 3f(x)\,dx =$ $3(5) = 15$

b. $\int_1^5 2f(x) - 6\,dx =$ $2(5) - 6(4) = -24 + 10 = -14$

c. $\int_3^3 g(x)\,dx =$ 0

d. $\int_1^5 f(x) - 2g(x)\,dx =$ $5 - 2(-3) = 11$

e. $\int_5^1 f(x)\,dx =$ -5

f. $\int_5^8 f(x)\,dx =$ $\int_1^8 f(x)\,dx - \int_1^5 f(x)\,dx = 8 - 5 = 3$

5. [3 points] Use geometry to determine $\int_{-4}^{0} \sqrt{16 - x^2}\,dx$. Justify your answer with a few words or a sketch.

$= \frac{1}{4}\pi(4^2) = \frac{1}{4} \cdot 16\pi = 4\pi.$